UNIVERSITY OF CAMBRIDGE
Faculty of Mathematics

SCHEDULES OF LECTURE COURSES

FOR THE MATHEMATICAL TRIPOS 2003-2004

This booklet is intended for Part IA students

IMPORTANT NOTICE: Schedules 2003/04

Important Notice

For this year only the Schedules booklet is being printed in three versions, one for Part IA students, one for Part IB students and one for Part II students. The reason for this is that the Tripos is in the middle of significant revisions and it seemed too complicated to include all the different versions in one booklet. This booklet contains the courses for Part IA (2003/04) and for Part IB (2004/05).

The Part IA courses have not been revised and there are at present no plans to do so for later years. However, there are no longer any designated 'pull-forward' courses (Pull-forward courses were Part IB courses designed to be taken by first year students).

One reason for abandoning the pull-forward system was that it was felt that many students were taking on extra courses at the expense of obtaining a thorough understanding of the Part IA courses. You are reminded that, if you find yourself with spare time that you would like to fill with mathematics beyond Part IA, you may attend any other lectures given by the Faculty (your Director of Studies will advise you about this); there are also excellent talks arranged by the various student mathematics societies; and there are plenty of books recommended in the schedules.

Some of the Part IB courses given in this booklet are new, some are modified versions of last year's courses and some are unchanged. Not all of the new schedules have been finalised.

There is also a change in the structure: there are no longer any 'O-courses'. These courses were essentially pull-forward courses from Part II(A). To compensate for the loss of the O-courses from the second year, the number of courses available in Part IB has been increased, offering the opportunity to take extra courses as well as a wider choice.

The Faculty Board has issued the following guideline regarding choice of course in Part IB:

Part IB of the Mathematical Tripos provides a wide range of courses from which students should, in consultation with their Directors of Studies, make a selection based on their individual interests and preferred work load, bearing in mind that it is better to do a number of courses thoroughly than to do many courses scrappily.

Major changes are planned for Part II, to be implemented in 2004/05. The full details are not yet agreed, but the aim is combine the two existing Alternatives while retaining different styles of course. This means that the choice of courses will be hugely widened.

SCHEDULES OF LECTURE COURSES
FOR THE MATHEMATICAL TRIPOS 2003-2004

Overview

Examinations

The form of each examination (number of questions in each section, number of questions which may be attempted, and range of subjects examined on each paper) is determined by the Faculty Board. Details are given later in this booklet in the *General Arrangements* sections for each part of the Tripos. The Faculty Board provides verbal classification criteria (see below) and recommends approximate percentages of candidates for each class (30% firsts, 40-45% upper seconds, 20-25% lower seconds, and not more than 6% thirds and below). Detailed marking schemes and and actual borderlines are decided by the examiners.

The Faculty Board of Mathematics have *recommended* to Examiners that, in addition to a numerical mark, extra credit should be available for the completeness and quality of each answer. An *alpha* quality mark signifies an answer of high quality which is substantially complete (normally at least three-quarters of the marks). A *beta* quality mark normally signifies at least half marks.

Quality marks as well as numerical marks are taken into account by the examiners in deciding the class borderlines. Examiners use a merit mark which takes into account raw mark, number of alphas and number of betas to produce an intial ordering of candidates. The formula for the merit mark may vary from year to year, in the light of experience, and is different for different parts of the Tripos. The merit mark is not a mechanistic formula for classing candidates; it is merely a guide to help the examiners draw up rough borderlines between the classes.

Very careful scrutiny is given to candidates near the borderlines and other factors besides marks and quality marks may be taken into account. The Faculty Board recommends that no distinction is made between marks obtained on the computer projects and marks obtained in the exam room, and no requirement is placed on the candidates to produce answers on a range of mathematical material beyond that imposed by the structure of the exam itself. When it is being decided whether a candidate should obtain honours (i.e. be placed in the third or higher class), individual considerations are always paramount and a merit mark borderline is not normally given. The approximate borderlines of last year's examinations are given for guidance in the *General Arrangements* section of each part of the Tripos.

Examinations are 'single-marked', but safety checks are made on all scripts to ensure that all work is marked and all marks are correctly added and transcribed. Scripts are identified only by candidate number until the final class-lists have been drawn up.

Calculators are not allowed in any paper of Parts IB and II of the Tripos, nor in Papers 1–4 of Part IA. Examination questions will be set in such a way as not to require the use of calculators. Calculators of the kind approved by the Board of Examinations (Casio fx 100D, Casio fx 115D (any version) or Casio fx 570 (any version)) are permitted in the Computer Science and Physics papers of Options (b) and (c) of Part IA.

Formula booklets are not permitted, but candidates will not be required to quote elaborate formulae from memory.

Classification Criteria

Candidates in each part of the undergraduate Tripos are classified into four classes: first class, upper second class (2.1), lower second class (2.2) and third class. Candidates who do not achieve the level required for a third class either get an allowance towards an ordinary degree (i.e. not an honours degree), or they fail. Such students cannot normally continue working for an honours degree in mathematics.

The Faculty Board recommends to examiners the following criteria for deciding the different classes.

The First Class

Candidates placed in the first class will have demonstrated a full command and secure understanding of a considerable range of examinable material. They will have presented standard arguments accurately, showing skill in applying their knowledge, and generally will have produced substantially correct solutions to questions.

The Upper Second Class

Candidates placed in the upper second class will have demonstrated good knowledge and understanding of a range of examinable material. They will have presented standard arguments accurately and will have shown some ability to apply their knowledge to solve problems. A fair number of their answers to both straightforward and more challenging (parts of) questions will have been substantially correct.

The Lower Second Class

Candidates placed in the lower second class will have demonstrated knowledge but limited understanding of some of the examinable material. They will have been aware of relevant mathematical issues, but their presentation of standard arguments will sometimes have been fragmentary or imperfect. They will have difficulty completing questions and little success in tackling more challenging problems: they will have produced substantially correct solutions to some straightforward (parts of) questions, but many of their attempts will have been good but incomplete.

The Third Class

Candidates placed in the third class will have demonstrated some knowledge but little understanding of the examinable material. They will have made reasonable attempts at a small number of questions, but will have lacked the skills to complete many of them. Generally their accounts of standard arguments will have been garbled and their approach to problems misguided and ineffective.

Ordinary or Fail

Candidates not classed 'with honours' will have demonstrated little or no knowledge of the examinable material. Their answers to questions will have been generally misguided or fragmentary, and their account of even straightforward bookwork will have been garbled or wrong. They will have shown little awareness of the mathematical issues involved in solving the problems set.

Data Protection Act 1998

To meet the University's obligations under the Data Protection Act 1998, the Faculty deals with data relating to individuals and their examination marks as follows:

- Marks for individual questions and Computational Projects are released routinely after the examinations.
- Data appearing on individual examination scripts are available on application to the University Data Protection Officer and on payment of a fee. Scripts are kept, in line with the University policy, for four months following the examinations and are then destroyed.

You should note that Examiners write little more on scripts than ticks, crosses, underlines and mark subtotals and totals.

Lectures

The schedule for each course is minimal for lecturing and maximal for examining. The topics starred in the schedules will be lectured, but questions will not be set on them in examinations. The numbers which appear in brackets at the end of subsections or paragraphs indicate the approximate number of lectures likely to be devoted to the subsection or paragraph concerned.

The Faculty Board revise the Schedules from time to time. The Secretary will be glad to receive comments for consideration by the Board.

Supervisions

Directors of Studies will arrange supervisions for each course as they think appropriate. Lecturers will hand out examples sheets which supervisors may use. According to Faculty Board guidelines, the number of examples sheets for 24-lecture, 16-lecture and 12-lecture courses should be 4, 3 and 2 respectively.

Feedback

Constructive feedback of all sorts and from all sources is welcomed by everyone concerned in providing courses for the Mathematical Tripos. There are many different feedback routes. Each lecturer hands out a questionnaire towards the end of the course and students are sent a combined e-mail questionnaire at the end of each year. These questionnaires are particularly important in shaping the future of the Tripos and the Faculty Board urge all students to respond. To give feedback during the course of the term, students can e-mail the Faculty Board and Teaching Committee students representatives and anyone can e-mail the anonymous rapid response Faculty 'hotline' `hotline@maths.cam.ac.uk`. Feedback can be sent directly to the Teaching Committee via the Faculty Office. Feedback on college-provided teaching can be given to Directors of Studies or Tutors at any time.

Student Representatives

There are two student representatives on each of the Faculty Board, the Teaching Committee and the Curriculum Committee, who are normally elected or appointed in January or February of each year. Their role is to advise the committees on the student point of view, to collect opinion from and liaise with the student body. They operate a website :

`http://www.damtp.cam.ac.uk/user/studrep`

and their email address is :

`student.reps@damtp.cam.ac.uk`.

Books

A list of books is given after each schedule. Books marked with † are particularly well suited to the course. Some of the books are out of print; these are retained on the list because they should be available in college libraries (as should all the books on the list) and may be found in second-hand bookshops.

Aims and Objectives

The **aims** of the Faculty for Parts IA, IB and II of the Mathematical Tripos are:

- to provide a challenging course in mathematics and its applications for a range of students that includes some of the best in the country;
- to provide a course that is suitable both for students aiming to pursue research and for students going into other careers;
- to provide an integrated system of teaching which can be tailored to the needs of individual students;
- to develop in students the capacity for learning and for clear logical thinking;
- to continue to attract and select students of outstanding quality;
- to produce the high calibre graduates in mathematics sought by employers in universities, the professions and the public services.
- to provide an intellectually stimulating environment in which students have the opportunity to develop their skills and enthusiasms to their full potential;
- to maintain the position of Cambridge as a leading centre, nationally and internationally, for teaching and research in mathematics.

The **objectives** of Parts IA, IB and II of the Mathematical Tripos are as follows:

After completing Part IA, students should have:

- made the transition in learning style and pace from school mathematics to university mathematics;
- been introduced to basic concepts in higher mathematics and their applications, including

  (i) the notions of proof, rigour and axiomatic development, (ii) the generalisation of familiar mathematics to unfamiliar contexts, (iii) the application of mathematics to problems outside mathematics;
- laid the foundations, in terms of knowledge and understanding, of tools, facts and techniques, to proceed to Part IB.

After completing Part IB, students should have:

- covered material from a range of pure mathematics, statistics and operations research, applied mathematics, theoretical physics and computational mathematics, and studied some of this material in depth;
- acquired a sufficiently broad and deep mathematical knowledge and understanding to enable them both to make an informed choice of courses in Part II and also to study these courses.

After completing Part II, students should have:

- developed the capacity for

  (i) solving both abstract and concrete problems, (ii) presenting a concise and logical argument, and (iii) (in most cases) using standard software to tackle mathematical problems;
- studied advanced material in the mathematical sciences, some of it in depth.

PART IA

GENERAL ARRANGEMENTS

Overview

There are three options: (*a*) Pure and Applied Mathematics; (*b*) Mathematics with Computer Science; and (*c*) Mathematics with Physics. Option (*a*) is intended primarily for students who expect to proceed to Part IB of the Mathematical Tripos, while Options (*b*) and (*c*) are intended primarily for those expecting to proceed to Part IB of the Computer Science Tripos or of the Natural Sciences Tripos. However, students are not debarred from taking the opposite choice in either case; in that event, reading in the Long Vacation following the examination may be recommended by their Director of Studies.

For Options (*b*) and (*c*), two of the lecture courses (Numbers and Sets, Dynamics) are replaced by courses from the Computer Science and Natural Sciences Tripos, respectively. Students wishing to examine the schedules for these courses should consult the documentation of the appropriate faculty, for example on http://www.cl.cam.ac.uk/UoCCL/teaching/ and http://www.phy.cam.ac.uk/teaching/.

The Faculty Board of Mathematics have given notice that the form of the examination for Part IA of the Mathematical Tripos will be as follows:

> Candidates taking Option (*a*) (Pure and Applied Mathematics) will take Papers 1, 2, 3 and 4 of Part IA of the Mathematical Tripos. Candidates taking Option (*b*) (Mathematics with Computer Science) will take Papers 1, 2 and 3 from the Mathematical Tripos and Paper 1 from Part IA of the Computer Science Tripos, and must submit a portfolio of assessed laboratory work. Candidates taking Option (*c*) (Mathematics with Physics) will take Papers 1, 2 and 3 from the Mathematical Tripos and the Physics paper from the Natural Sciences Tripos, and must submit practical notebooks.
>
> Papers 1, 2, 3 and 4 from the Mathematical Tripos will be divided into two Sections. There will be four questions in Section I and eight questions in Section II. Candidates may attempt all the questions in Section I and at most five questions from Section II, of which no more than three may be on the same course. Each question in Section II will carry twice the weight of each question in Section I.
>
> There will be two Section I questions and four Section II questions per 24 lectures. Each section of each paper is divided equally between two courses:

Paper 1:	Algebra and Geometry, Analysis I
Paper 2:	Differential Equations, Probability
Paper 3:	Algebra and Geometry, Vector Calculus
Paper 4:	Numbers and Sets, Dynamics.

Candidates are reminded that little credit is given in examinations for fragmentary answers.

Candidates are also reminded that calculators are not permitted for Papers 1, 2, 3 and 4 of the Mathematical Tripos. Calculators of the kind approved by the Board of Examinations, Casio fx100D, Casio fx115D or Casio fx570 (which must be marked), are permitted in the Computer Science and Physics papers.

On Papers 1, 2, 3 and 4, Section I questions are marked out of 10, one alpha or one beta being available, and Section II questions are marked out of 20, one double alpha or one double beta being available.

For Mathematics with Computer Science or Mathematics with Physics candidates, the marks and quality marks for the Computer Science or Physics paper are scaled to bring them in line with Paper 4.

Last year, the examiners used the following formula to obtain the merit mark (M) in terms of the number of raw marks (m), the number of alphas (α) and the number of betas (β):

$$M = \begin{cases} 2m + 10\alpha + 3\beta - 60 & \text{for } \alpha \geq 20 \\ 2m + 7\alpha + 3\beta & \text{for } \alpha < 20 \end{cases}$$

The following table shows the merit mark used by the examiners for initial borderlines (see page 4) and the marks, alphas and betas of a typical candidate placed near the bottom of each class. At the lowest borderlines, individual consideration of candidates is always paramount, so no initial borderline is used and there are no typical candidates; *very few* candidates fail to achieve at least a third class.

Borderline	Merit mark	Borderline candidate
1/2(i)	950	344,29,n/a
2(i)/2(ii)	670	265,17,16
2(ii)/3	500	200,10,10

ALGEBRA AND GEOMETRY **48 lectures, Michaelmas term**

Review of complex numbers, modulus, argument and de Moivre's theorem. Informal treatment of complex logarithm, *n-th* roots and complex powers. Equations of circles and straight lines. Examples of Möbius transformations. [3]

Vectors in R^3. Elementary algebra of scalars and vectors. Scalar and vector products, including triple products. Geometrical interpretation. Cartesian coordinates; plane, spherical and cylindrical polar coordinates. Suffix notation: including summation convention and δ_{ij}, ϵ_{ijk}. [5]

Vector equations. Lines, planes, spheres, cones and conic sections. Maps: isometries and inversions. [2]

Introduction to $\mathbb{R}^n$, scalar product, Cauchy–Schwarz inequality and distance. Subspaces, brief introduction to spanning sets and dimension. [4]

Linear maps from $\mathbb{R}^m$ to $\mathbb{R}^n$ with emphasis on $m, n \leqslant 3$. Examples of geometrical actions (reflections, dilations, shears, rotations). Composition of linear maps. Bases, representation of linear maps by matrices, the algebra of matrices. [5]

Determinants, non-singular matrices and inverses. Solution and geometric interpretation of simultaneous linear equations (3 equations in 3 unknowns). Gaussian Elimination. [3]

Discussion of $\mathbb{C}^n$, linear maps and matrices. Eigenvalues, the fundamental theorem of algebra (statement only), and its implication for the existence of eigenvalues. Eigenvectors, geometric significance as invariant lines. [3]

Discussion of diagonalization, examples of matrices that cannot be diagonalized. A real 3×3 orthogonal matrix has a real eigenvalue. Real symmetric matrices, proof that eigenvalues are real, and that distinct eigenvalues give orthogonal basis of eigenvectors. Brief discussion of quadratic forms, conics and their classification. Canonical forms for 2×2 matrices; discussion of relation between eigenvalues of a matrix and fixed points of the corresponding Möbius map. [5]

Axioms for groups; subgroups and group actions. Orbits, stabilizers, cosets and conjugate subgroups. Orbit-stabilizer theorem. Lagrange's theorem. Examples from geometry, including the Euclidean groups, symmetry groups of regular polygons, cube and tetrahedron. The Möbius group; cross-ratios, preservation of circles, informal treatment of the point at infinity. [11]

Isomorphisms and homomorphisms of abstract groups, the kernel of a homomorphism. Examples. Introduction to normal subgroups, quotient groups and the isomorphism theorem. Permutations, cycles and transpositions. The sign of a permutation. [5]

Examples (only) of matrix groups; for example, the general and special linear groups, the orthogonal and special orthogonal groups, unitary groups, the Lorentz groups, quaternions and Pauli spin matrices. [2]

Appropriate books

M.A. Armstrong *Groups and Symmetry.* Springer–Verlag 1988 (£33.00 hardback).
D.M. Bloom *Linear Algebra and Geometry.* Cambridge University Press 1979 (out of print).
D.E. Bourne and P.C. Kendall *Vector Analysis and Cartesian Tensors.* Nelson Thornes 1992 (£30.75 paperback).
R.P. Burn *Groups, a Path to Geometry.* Cambridge University Press 1987 (£20.95 paperback).
J.A. Green *Sets and Groups: a first course in Algebra.* Chapman and Hall/CRC 1988 (£38.99 paperback).
E. Sernesi *Linear Algebra: A Geometric Approach.* CRC Press 1993 (£38.99 paperback).
D. Smart *Linear Algebra and Geometry.* Cambridge University Press 1988 (out of print).

NUMBERS AND SETS — 24 Lectures, Michaelmas term

[Note that this course is omitted from Part IA, Options (b) and (c).]

Introduction to numbers systems and logic

Overview of the natural numbers, integers, real numbers, rational and irrational numbers, algebraic and transcendental numbers. Brief discussion of complex numbers; statement of the Fundamental Theorem of Algebra.

Ideas of axiomatic systems and proof within mathematics; the need for proof; the role of counter-examples in mathematics. Elementary logic; implication and negation; examples of negation of compound statements. Proof by contradiction. [2]

Sets, relations and functions

Union, intersection and equality of sets. Indicator (characteristic) functions; their use in establishing set identities. Functions; injections, surjections and bijections. Relations, and equivalence relations. Counting the combinations or permutations of a set. The Inclusion-Exclusion Principle. [4]

The integers

The natural numbers: the well-ordering principle, and its equivalence with Mathematical Induction. Examples, including the Binomial Theorem. [2]

Elementary number theory

Prime numbers: existence and uniqueness of prime factorisation into primes; highest common factors and least common multiples. Euclid's proof of the infinity of primes. Euclid's algorithm. Solution in integers of ax+by = c.

Modular arithmetic (congruences). Units modulo n. Chinese Remainder Theorem. Wilson's Theorem; the Fermat-Euler Theorem. Public key cryptography and the RSA algorithm. [10]

The real numbers

Irrational numbers, including $\sqrt{2}$ and e. Decimal expansions. Construction of a transcendental number. [2]

Countability and uncountability

Definitions of finite, infinite, countable and uncountable sets. A countable union of countable sets is countable. Uncountability of $\mathbb{R}$. Non-existence of a bijection from a set to its power set. Indirect proof of existence of transcendental numbers. [4]

Appropriate books

R.B.J.T. Allenby *Numbers and Proofs.* Butterworth-Heinemann 1997 (£19.50 paperback).

R.P. Burn *Numbers and Functions: steps into analysis.* Cambridge University Press 2000 (£21.95 paperback).

H. Davenport *The Higher Arithmetic.* Cambridge University Press 1999 (£19.95 paperback).

A.G. Hamilton *Numbers, sets and axioms: the apparatus of mathematics.* Cambridge University Press 1983 (£20.95 paperback).

C. Schumacher *Chapter Zero: Fundamental Notions of Abstract Mathematics.* Addison-Wesley 2001 (£42.95 hardback).

I. Stewart and D. Tall *The Foundations of Mathematics.* Oxford University Press 1977 (£22.50 paperback).

DIFFERENTIAL EQUATIONS **24 lectures, Michaelmas term**

Basic calculus
Differentiation as a limit, the chain rule, Leibnitz' rule, elementary treatment of Taylor series; integration as an area, integration by substitution and parts; fundamental theorem of calculus; differentiation under integrals. [3]

First-order equations
Linear equations with constant coefficients: exponential growth, comparison with discrete equations, series solution; modelling examples including radioactive decay and time delay equation. Linear equations with non-constant coefficients: solution by integrating factor, series solution, comparison with discrete equations. [3]

Nonlinear equations: separable equations, families of solutions, isoclines, the idea of a flow and connection with vector fields, equilibrium solutions, stability by perturbation and phase-plane analysis; examples, including logistic equation and chemical kinetics; comparison with discrete equations including the logistic equation. [5]

Higher-order equations
Linear equations: complementary function and particular integral, linear independence, Wronskian, equations with constant coefficients and examples including radioactive sequences, comparison in simple cases with difference equations, reduction of order, resonance, transients, damping, response to step and impulse function inputs, coupled first order systems, series solutions including statement only of the need for the logarithmic solution. [8]

Nonlinear equations
Elementary phase-plane analysis, equilibrium and stability, examples including predator-prey systems. [5]

Appropriate books

W.E. Boyce and R.C. DiPrima *Elementary Differential Equations and Boundary-Value Problems.* Wiley 7th edition 2001 (£34.95 hardback). 8th ed. due for publication in May 2004

D.N. Burghes and M.S. Borrie *Modelling with Differential Equations.* Ellis Horwood 1981 (out of print).

W. Cox *Ordinary Differential Equations.* Butterworth-Heinemann 1996 (£14.99 paperback).

F. Diacu *An introduction to Differential Equations: Order and Chaos.* Freeman 2000 (£38.99 hardback).

N. Finizio and G. Ladas *Ordinary Differential Equations with Modern Applications.* Wadsworth 1989 (out of print).

D. Lomen and D. Lovelock *Differential Equations: Graphics-Models-Data.* Wiley 1999 (£80.95 hardback).

R.E. O'Malley *Thinking about Ordinary Differential Equations.* Cambridge University Press 1997 (£19.95 paperback).

D.G. Zill and M.R. Cullen *Differential Equations with Boundary Value Problems.* Brooks/Cole 2001 (£37.00 hardback).

ANALYSIS I **24 lectures, Lent term**

Limits and convergence
Limit of sequences in $\mathbb{R}$ and $\mathbb{C}$. Sums, products and quotients. Axiom that bounded monotonic sequences in $\mathbb{R}$ converge. Convergent series, linear combinations of series. Absolute convergence; absolute convergence implies convergence. Comparison and ratio tests, alternating series test. [5]

Least upper bounds
Least upper bounds and greatest lower bounds; simple examples. Least upper bound axiom and its equivalence with convergence of bounded monotonic sequences. Applications. [3]

Continuity
Continuity of real- and complex-valued functions defined on subsets of $\mathbb{R}$ and $\mathbb{C}$. The intermediate value theorem. The Bolzano-Weierstrass theorem. A continuous function on a closed bounded interval is bounded and attains its bounds. [3]

Differentiability
Differentiability of functions from $\mathbb{R}$ to $\mathbb{R}$. Derivative of sums and products. The chain rule. Derivative of the inverse function. Rolle's theorem; the mean value theorem. One-dimensional version of the inverse function theorem. Taylor's theorem from $\mathbb{R}$ to $\mathbb{R}$; Lagrange's form of the remainder. Complex differentiation. Taylor's theorem from $\mathbb{C}$ to $\mathbb{C}$ (statement only). [4]

Power series
Complex power series and radius of convergence. Exponential, trigonometric and hyperbolic functions, and relations between them. *Direct proof of the differentiability of a power series within its circle of convergence*. [4]

Integration
Definition and basic properties of the Riemann integral. A non-integrable function. Integrability of monotonic functions. Integrability of piecewise-continuous functions (statement only). The fundamental theorem of calculus. Differentiation of indefinite integrals. Integration by parts. The integral form of the remainder in Taylor's theorem. Improper integrals. [5]

Appropriate books

H. Anton *Calculus.* Wiley 2002 (£41.71 paperback with CD-ROM).
T.M. Apostol *Calculus.* Wiley 1967-69 (£181.00 hardback).
J.C. Burkill *A First Course in Mathematical Analysis.* Cambridge University Press 1978 (£18.95 paperback).
J.B. Reade *Introduction to Mathematical Analysis.* Oxford University Press (out of print).
M. Spivak *Calculus.* Addison–Wesley/Benjamin–Cummings 1967 (out of print).

VECTOR CALCULUS **24 lectures. Lent term**

Partial differentiation

Partial derivatives, geometrical interpretation, statement (only) of symmetry of mixed partial derivatives, chain rule. Directional derivatives. The gradient of a real-valued function, its interpretation as normal to level surfaces (examples including the use of cylindrical, spherical *and general orthogonal curvilinear* coordinates). Statement of Taylor series for functions on $\mathbb{R}^n$. Local extrema of real functions, classification using the Hessian matrix. [6]

Curves, line integrals and differentials

Parametrised curves and arc length, tangents and normals to curves in $\mathbb{R}^3$, the radius of curvature. Line integrals, conservative fields. Informal treatment of differentials, exact differentials. [3]

Integration in $\mathbb{R}^2$ and $\mathbb{R}^3$

Surface and volume integrals. Determinants, Jacobians, and change of variables. [4]

Divergence, curl and ∇^2 in Cartesian coordinates, examples; formulae for these operators (statement only) in cylindrical, spherical *and general orthogonal curvilinear* coordinates. Vector derivative identities. The divergence theorem, Green's theorem, Stokes' theorem. Irrotational fields. [6]

Laplace's equation

Laplace's equation in $\mathbb{R}^2$ and $\mathbb{R}^3$. Examples of solutions. Green's (second) theorem. Bounded regions and Dirichlet boundary condition; uniqueness and maximum principle. [3]

Fundamental solutions in $\mathbb{R}^2$ and $\mathbb{R}^3$ with point sources. Poisson's equation and its solution, interpretation in electrostatics and gravity. [2]

Appropriate books

H. Anton *Calculus.* Wiley Student Edition 2000 (£33.95 hardback).

T.M. Apostol *Calculus.* Wiley Student Edition 1975 (Vol. II £37.95 hardback).

M.L. Boas *Mathematical Methods in the Physical Sciences.* Wiley 1983 (£32.50 paperback).

† D.E. Bourne and P.C. Kendall *Vector Analysis and Cartesian Tensors.* 3rd edition, Nelson Thornes 1999 (£29.99 paperback).

E. Kreyszig *Advanced Engineering Mathematics.* Wiley International Edition 1999 (£30.95 paperback, £97.50 hardback).

J.E. Marsden and A.J.Tromba *Vector Calculus.* Freeman 1996 (£35.99 hardback).

P.C. Matthews *Vector Calculus.* SUMS (Springer Undergraduate Mathematics Series) 1998 (£18.00 paperback).

† K. F. Riley, M.P. Hobson, and S.J. Bence *Mathematical Methods for Physics and Engineering.* Cambridge University Press 2002 (£27.95 paperback, £75.00 hardback).

H.M. Schey *Div, grad, curl and all that: an informal text on vector calculus.* Norton 1996 (£16.99 paperback).

M.R. Spiegel *Schaum's outline of Vector Analysis.* McGraw Hill 1974 (£16.99 paperback).

PROBABILITY **24 lectures, Lent term**

Basic concepts
Classical probability, equally likely outcomes. Combinatorial analysis, permutations and combinations. Stirling's formula (asymptotics for $\log n!$ proved). [3]

Axiomatic approach
Axioms (countable case). Probability spaces. Addition theorem, inclusion-exclusion formula. Boole and Bonferroni inequalities. Independence. Binomial, Poisson and geometric distributions. Relation between Poisson and binomial distributions. Conditional probability, Bayes's formula. Examples, including Simpson's paradox. [5]

Discrete random variables
Expectation. Functions of a random variable, indicator function, variance, standard deviation. Covariance, independence of random variables. Generating functions: sums of independent random variables, random sum formula, moments.

Conditional expectation. Random walks: gambler's ruin, recurrence relations. Difference equations and their solution. Mean time to absorption. Branching processes: generating functions and extinction probability. Combinatorial applications of generating functions. [7]

Continuous random variables
Distributions and density functions. Expectations; expectation of a function of a random variable. Uniform, normal and exponential random variables. Memoryless property of exponential distribution.

Joint distributions: transformation of random variables, examples. Simulation: generating continuous random variables, independent normal random variables. Geometrical probability: Bertrand's paradox, Buffon's needle. Correlation coefficient, bivariate normal random variables. [6]

Inequalities and limits
Markov's inequality, Chebyshev's inequality. Weak law of large numbers. Convexity: Jensen's inequality, AM/GM inequality.

Moment generating functions. Statement of central limit theorem and sketch of proof. Examples, including sampling. [3]

Appropriate books

W. Feller *An Introduction to Probability Theory and its Applications, Vol. I.* Wiley 1968 (£73.95 hardback).

† G. Grimmett and D. Welsh *Probability: An Introduction.* Oxford University Press 1986 (£23.95 paperback).

† S. Ross *A First Course in Probability.* Wiley 2002 (£39.99 paperback).

D.R. Stirzaker *Elementary Probability.* Cambridge University Press 1994 (£19.95 paperback). 2nd ed. due for publication in October 2003

DYNAMICS **24 lectures, Lent term**

[Note that this course is omitted from Part IA, Options (b) and (c).]

Basic concepts: motion, reference frames and units; acceleration; Newton's laws, mass and force; Galilean transformation; forces including gravity, electrostatic, Lorentz, elastic, friction (dry, linear, quadratic) and constraining. [2]

Dimensional analysis, scaling and non-dimensionalisation (e.g. use of dimensionless parameters in two force problems). [1]

Examples: circular orbits, electron in a uniform electromagnetic field, simple harmonic motion (free, damped, forced), projectiles with quadratic friction, varying mass (e.g. avalanche, rockets), constrained motion (including period of finite amplitude pendulum). [4]

Work: kinetic energy, potential energy in one dimension and in three dimensions for conservative forces. Motion and the shape of the potential energy function; stability of equilibria, escape velocity. Small amplitude oscillations in one dimension. [3]

Impulse and collisions (elastic and inelastic): centre of mass, energy, coefficient of restitution. [1]

Rotational motion, acceleration in plane polar coordinates. Angular velocity, angular momentum in three dimensions. Rotating frames; apparent gravity, centrifugal and Coriolis accelerations. Foucault pendulum. [3]

Orbits: gravitational potential for a sphere; escape velocity; shape of the orbit for $1/r^2$ forces; orbital equations in terms of $u(\theta)$; period; Kepler's laws; ∗interplanetary travel∗; stability of circular orbits. Rutherford scattering, impact parameter and cross-section. [4]

Systems of particles: momentum, kinetic energy and angular momentum about centre of mass; reduced mass in 2-body problem. [1]

Rotation of rigid bodies. Moments of inertia. Simple examples of rotation and translation with a constant axis; centre of percussion (e.g. impulse on a cricket bat). [3]

Application of phase-plane techniques; classification of equilibria; conservative system; damped systems: ∗limit cycles∗. Forced pendulum, ∗hysteresis, chaos∗. [2]

Appropriate books

† V.D. Barger and M.G. Olsson *Classical Mechanics: a modern perspective.* McGraw-Hill 1995 (out of print).

D.N. Burghes and A.M. Downs *Modern Introduction to Classical Mechanics and Control.* Ellis Horwood 1975 (out of print).

C.D. Collinson *Introductory Mechanics.* Arnold 1980 (out of print).

† A.P. French and M.G. Ebison *Introduction to Classical Mechanics.* Kluwer 1986 (£33.25 paperback).

M. A. Lunn *A First Course in Mechanics.* Oxford University Press 1991 (£17.50 paperback).

PHYSICS (non-examinable) **12 lectures, Michaelmas term**

Idea of mass and force, Newton's law $F = ma$. Motion of projectiles. Simple pendulum. Circular motion, angular velocity. Examples of simple mechanics problems. Hooke's law.

Boyle's law, pressure, temperature, ideal gases. Heat as a form of energy.

Electric currents, voltage, Ohm's law. Simple circuits, resistances in parallel and series. Capacitors and inductances in circuits. Electric and magnetic fields.

Elementary ideas in optics, Snell's law, Fermat's principle. Light as a wave motion, interference, polarization. Infra-red and ultra-violet radiation, radio waves, the electromagnetic theory of light and the electromagnetic spectrum.

Structure of atoms, very simple treatment of hydrogen atom using quantisation of angular momentum. Bohr radius as characteristic size of atoms.

PART IB, 2004/05

GENERAL ARRANGEMENTS

Overview

Seventeen courses, including Computational Projects, are examined in Part IB.

The lectures for Computational Projects will normally be attended in the Easter term of the first year, the Computational Projects themselves being done in the Michaelmas and Lent Terms of the second year (or in the preceding Long Vacation).

Some courses are lectured twice, to give the timetable greater flexibility: Quantum Mechanics and Fluid Dynamics are lectured in Michaelmas and Lent terms; Special Relativity is lectured in Easter term and Lent term.

Complex variable is presented in two versions: Complex Methods amd Complex Analysis. There is considerable overlap in material between these courses, but Complex Methods is presented from an applied point of view and Complex Analysis is presented from a pure point of view with greater concentration on proof.

Two courses (Optimisation and Numerical Analysis) can be taken in the Easter term either of the first year or of the second year.

Metric and Topological Spaces can also be taken in either Easter term, but it should be noted that some material will prove useful for Complex Analysis.

The Faculty Board guidance reagarding choice of courses in Part IB is as follows:

Part IB of the Mathematical Tripos provides a wide range of courses from which students should, in consultation with their Directors of Studies, make a selection based on their individual interests and preferred work load, bearing in mind that it is better to do a number of courses thoroughly than to do many courses scrappily.

LINEAR ALGEBRA **24 lectures, Michaelmas term**

Definition of a vector space (over $\mathbb{R}$ or $\mathbb{C}$), subspaces, the space spanned by a subset. Linear independence, bases, dimension. Direct sums and complementary subspaces. [3]

Linear maps, isomorphisms. Relation between rank and nullity. The space of linear maps from U to V, representation by matrices. Change of basis. Row rank and column rank. [4]

Determinant and trace of a square matrix. Determinant of a product of two matrices and of the inverse matrix. Determinant of an endomorphism. The adjugate matrix. [3]

Eigenvalues and eigenvectors. Diagonal and triangular forms. Characteristic and minimal polynomials. Cayley-Hamilton Theorem over $\mathbb{C}$. Algebraic and geometric multiplicity of eigenvalues. Statement and illustration of Jordan normal form. [4]

Dual of a finite-dimensional vector space, dual bases and maps. Matrix representation, rank and determinant of dual map [2]

Bilinear forms. Matrix representation, change of basis. Symmetric forms and their link with quadratic forms. Diagonalisation of quadratic forms. Law of inertia, classification by rank and signature. Complex Hermitian forms. [4]

Inner product spaces, orthonormal sets, orthogonal projection, $V = W \oplus W^{\perp}$. Gram-Schmidt orthogonalisation. Adjoints. Diagonalisation of Hermitian matrices. Orthogonality of eigenvectors and properties of eigenvalues. [4]

Appropriate books

C.W. Curtis *Linear Algebra: an introductory approach.* Springer 1984 (£38.50 hardback).
P.R. Halmos *Finite-dimensional vector spaces.* Springer 1993 (£31.50 hardback).
K. Hoffman and R. Kunze *Linear Algebra.* Prentice-Hall 1971 (£72.99 hardback).

GROUPS, RINGS AND MODULES **24 lectures, Lent term**

Groups

Basic concepts of group theory recalled from Algebra and Geometry. Normal subgroups, quotient groups and isomorphism theorems. Permutation groups. Groups acting on sets, permutation representations. Conjugacy classes, centralizers and normalizers. The centre of a group. Elementary properties of finite p-groups. Examples of finite linear groups and groups arising from geometry. Simplicity of A_5.

Sylow subgroups and Sylow theorems. Applications, groups of small order. [7]

Rings

Definition and examples of rings (commutative, with 1). Ideals, homomorphisms, quotient rings, isomorphism theorems. Prime and maximal ideals. Fields. The characteristic of a field. Field of fractions of an integral domain.

Factorization in rings; units, primes and irreducibles. Unique factorization in principal ideal domains, and in polynomial rings. Gauss' Lemma and Eisenstein's irreducibility criterion.

Rings $\mathbb{Z}[\alpha]$ of algebraic integers as subsets of $\mathbb{C}$ and quotients of $\mathbb{Z}[x]$. Examples of Euclidean domains and uniqueness and non-uniqueness of factorization. Factorization in the ring of Gaussian integers; representation of integers as sums of two squares.

Ideals in polynomial rings. Hilbert basis theorem. [9]

Modules

Definitions, examples of vector spaces, abelian groups and vector spaces with an endomorphism. Submodules, homomorphisms, quotient modules and direct sums. Equivalence of matrices, canonical form. Structure of finitely generated modules over Euclidean domains, applications to abelian groups and Jordan normal form. [8]

Appropriate books

P.M.Cohn *Classic Algebra.* Wiley, 2000 (£29.95 paperback).

J.B. Fraleigh *A First Course in abstract Algebra.* Addison Wesley, 2003 (£47.99 paperback).

B. Hartley and T.O. Hawkes *Rings, Modules and Linear Algebra: a further course in algebra.* Chapman and Hall, 1970 (out of print).

I. Herstein *Topics in Algebra.* John Wiley and Sons, 1975 (£45.99 hardback).

P.M. Neumann, G.A. Stoy and E.C. Thomson *Groups and Geometry.* OUP 1994 (£35.99 paperback).

M. Artin *Algebra.* Prentice Hall, 1991 (£53.99 hardback).

ANALYSIS II **24 lectures, Michaelmas term**

Uniform convergence

The general principle of uniform convergence. A uniform limit of continuous functions is continuous. Uniform convergence and termwise integration and differentiation of series of real-valued functions. Local uniform convergence of power series. [3]

Uniform continuity and integration

Continuous functions on closed bounded intervals are uniformly continuous. Review of basic facts on Riemann integration (from Analysis I); proof that piecewise-continuous functions are integrable. Informal discussion of integration of complex-valued and $\mathbb{R}^n$-valued functions of one variable; proof that $\| \int_a^b f(x)\, dx\| \leq \int_a^b \|f(x)\|\, dx$. [2]

$\mathbb{R}^n$ as a normed space

Definition of a normed space. Examples, including the Euclidean norm on $\mathbb{R}^n$ and the uniform norm on $\mathcal{C}[a,b]$. Lipschitz mappings and Lipschitz equivalence of norms. The Bolzano–Weierstrass theorem in $\mathbb{R}^n$. Completeness. Open and closed sets. Continuity for functions between normed spaces. A continuous function on a closed bounded set in $\mathbb{R}^n$ is uniformly continuous and has closed bounded image. All norms on a finite-dimensional space are Lipschitz equivalent. [5]

Differentiation from $\mathbb{R}^m$ to $\mathbb{R}^n$

Definition of derivative as a linear map; elementary properties, the chain rule. Partial derivatives; continuous partial derivatives imply differentiability. Higher-order derivatives; symmetry of mixed partial derivatives (assumed continuous). Taylor's theorem. The mean value inequality. Path-connectedness for subsets of $\mathbb{R}^n$; a function having zero derivative on a path-connected open subset is constant. [6]

Metric spaces

Definition and examples. *Metrics used in Geometry*. Limits, continuity, balls, neighbourhoods, open and closed sets. [4]

The Contraction Mapping Theorem

The contraction mapping theorem. Applications including the inverse function theorem (proof of continuity of inverse function, statement of differentiability). Picard's solution of differential equations. [4]

Appropriate books

† J.C. Burkill and H. Burkill *A Second Course in Mathematical Analysis.* Cambridge University Press 2002 (£29.95 paperback).

A.F. Beardon *Limits: a new approach to real analysis.* Springer 1997 (£22.50 hardback).

† W. Rudin *Principles of Mathematical Analysis.* McGraw–Hill 1976 (£35.99 paperback).

W.A. Sutherland *Introduction to Metric and Topological Space.* Clarendon 1975 (£21.00 paperback).

A.J. White *Real Analysis: An Introduction.* Addison–Wesley 1968 (out of print).

T.W. Körner *A companion to analysis.* (Unpublished manuscript available from www.dpmms.cam.ac.uk/ twk until 30th November 2003. After this, the book should be published by the AMS.).

METRIC AND TOPOLOGICAL SPACES — 12 lectures, Easter term

Metrics
Definition and examples. Limits and continuity. Open sets and neighbourhoods. Characterizing limits and continuity using neighbourhoods and open sets. [3]

Topology
Definition of a topology. Metric topologies. Further examples. Neighbourhoods, closed sets, convergence and continuity. Hausdorff spaces. Homeomorphisms. Topological and non-topological properties. Completeness. Subspace, quotient and product topologies. [3]

Connectedness
Definition using open sets and integer-valued functions. Examples, including intervals. Components. The continuous image of a connected space is connected. Path-conectedness. Path-connected spaces are connected but not conversely. Connected open sets in Euclidean space are path-connected. [4]

Compactness
Definition using open covers. Examples: Finite sets and $[0,1]$. Closed subsets of compact spaces are compact. Compact subsets of a Hausdorff space must be closed. The compact subsets of the real line. Continuous images of compact sets are compact. Quotient spaces. Continuous real-valued functions on a compact space are bounded and attain their bounds. The product of two compact spaces is compact. The compact subsets of Eudlidean space. Sequential compactness. [4]

Appropriate books

W.A. Sutherland *Introduction to metric and topological space.* Clarendon 1975 (£21.00 paperback).

A.J. White *Real analysis: an introduction.* Addison-Wesley 1968 (out of print).

GEOMETRY **16 lectures, Lent Term**

Schedule to be announced

COMPLEX ANALYSIS **16 lectures, Lent term**

Analytic functions
Complex differentiation and the Cauchy-Riemann equations. Examples. Conformal mappings. Informal discussion of branch points, examples of $\log z$ and z^c. [3]

Contour integration and Cauchy's theorem
Contour integration (for piecewise continuously differentiable curves). Statement and proof of Cauchy's theorem for star domains. Cauchy's integral formula, maximum modulus theorem, Liouville's theorem, fundamental theorem of algebra. Morera's theorem. [4]

Expansions and singularities
Uniform convergence of analytic functions; local uniform convergence. Differentiability of a power series. Taylor and Laurent expansions. Principle of isolated zeros. Residue at an isolated singularity. Classification of isolated singularities. [4]

The residue theorem
Winding numbers. Residue theorem. Jordan's lemma. Evaluation of definite integrals by contour integration. Rouché's theorem, principle of the argument. Open mapping theorem. [3]

Fourier transforms
Definition of Fourier transform. Examples of computation. Discussion of Fourier tranforms of convolutions and derivatives, and the inversion formula. [2]

Appropriate books

L.V. Ahlfors *Complex Analysis.* McGraw–Hill 1978 ().
† A.F. Beardon *Complex Analysis.* Wiley (out of print).
† G.J.O. Jameson *A First Course on Complex Functions.* Chapman and Hall (out of print).
H.A. Priestly *Introduction to Complex Analysis.* Oxford University Press 1990 ().
I. Stewart and D. Tall *Complex Analysis.* Cambridge University Press 1983 ().

COMPLEX METHODS **16 lectures, Lent term**

Analytic functions
Definition of an analytic function. Cauchy-Riemann equations. Analytic functions as conformal mappings; examples. Application to the solutions of Laplace's equation in various domains. Discussion of $\log z$ and z^a. [3]

Contour integration and Cauchy's Theorem
[*Proofs of theorems in this section will not be examined in this course.*]
Domains, contours, contour integrals. Cauchy's theorem and Cauchy's integral formula. Taylor and Laurent series. Zeros, poles and essential singularities. [3]

Residue calculus
Cauchy's residue theorem, calculus of residues. Jordan's lemma. Evaluation of definite integrals by contour integration. [5]

Fourier transforms
Definition and simple properties. The inversion formula. Applications to differential equations. Computation of transforms by contour integration. Transforms of step and delta functions.

Convolution. Parseval's formula. Application to time-invariant linear 'input' and 'output' systems. Causality. Frequency analysis of signals. [5]

Appropriate books

M.J. Ablowitz A.S. Fokas *Complex variables: introduction and applications.* CUP 2003 (£65.00).
G. Arfken and H. Weber *Mathematical Methods for Physicists.* Harcourt Academic 2001 (£38.95 paperback).
G. J. O. Jameson *A First Course in Complex Functions.* Chapman and Hall 1970 (out of print).
T. Needham *Visual complex analysis.* Clarendon 1998 (£28.50 paperback).
† H.A. Priestley *Introduction to Complex Analysis.* Clarendon 1990 (out of print).
† I. Stewart and D. Tall *Complex Analysis (the hitchhiker's guide to the plane).* Cambridge University Press 1983 (£24.95 paperback).

METHODS **24 lectures, Michaelmas term**

Fourier series and the wave equation
Periodic functions. Fourier series. Parseval's theorem. Wave equation for a string. Separation of variables. Normal modes for a string of finite length. Energy. Solution of form $f(x + ct) + g(x - ct)$. Wave reflection and transmission. [4]

Ordinary differential equations
Equations of the second order; initial value problems and problems with two fixed end points; solution using Green's function: notion of Green's function as an inverse operator. The Sturm–Liouville equation; eigenfunction and eigenvalues: reality of eigenvalues and orthogonality of eigenfunctions; eigenfunctions expansions (Fourier series as prototype), approximation in mean square, statement of completeness; expansion of δ-function and Green's function. [6]

Laplace's equation
Separation of variables in Cartesians, plane polar and spherical polar co-ordinates, solutions in spherical polars for axisymmetric systems only. Legendre's equation and solutions as Legendre polynomials (statement only); orthogonality; forms of P_0, P_1 and P_2. [3]

Calculus of variations
Stationary points of $f(x_1, \ldots, x_n)$: necessary and sufficient conditions for the free case; Lagrange multipliers, necessary conditions for the constrained case. Euler–Lagrange equations, functional derivatives, first integrals; use of Lagrange multipliers (statement only). Examples e.g. Fermat's principle. [5]

Cartesian tensors in $\mathbb{R}^3$
Tensor transformation laws, addition, multiplication, contraction, the quotient theorem. Isotropic second and third rank tensors. Symmetric and antisymmetric tensors. Revision of principal axes and diagonalization. Examples e.g. inertia, stress, conductivity. [6]

Appropriate books

G. Arfken and H. Weber *Mathematical Methods for Physicists.* Harcourt Academic 2001 (£38.95 hardback).
M.L. Boas *Mathematical Methods in the Physical Sciences.* Wiley 1983 (£34.50 hardback).
H. Jeffreys *Cartesian Tensors.* Cambridge University Press 1931 (out of print).
H. Jeffreys and B.S. Jeffreys *Methods of Mathematical Physics.* Cambridge University Press 1999 (£25.95 paperback).
J. Mathews and R.L. Walker *Mathematical Methods of Physics.* Benjamin/Cummings 1970 (£68.99 hardback).
K. F. Riley, M. P. Hobson, and S.J. Bence *Mathematical Methods for Physics and Engineering: a comprehensive guide.* Cambridge University Press 2002 (£29.95 paperback).
B. Spain *Tensor Calculus.* Dover 2003 (£8.95 paperback).

QUANTUM MECHANICS **16 lectures, Michaelmas and Lent terms**

Physical background

Photoelectric effect. Electrons in atoms and line spectra. Particle diffraction. [1]

Schrödinger equation and solutions

De Broglie waves. Schrödinger equation. Superposition principle. Probability interpretation, density and current. [2]

Stationary states. Free particle, Gaussian wave packet. Motion in 1-dimensional potentials, parity. Potential step, square well and barrier. Harmonic oscillator. [4]

Observables and expectation values

Position and momentum operators and expectation values. Canonical commutation relations. Uncertainty principle. [2]

Observables and Hermitian operators. Eigenvalues and eigenfunctions. Formula for expectation value. [2]

Hydrogen atom

Spherically symmetric wave functions for spherical well and hydrogen atom.

Orbital angular momentum operators. General solution of hydrogen atom. [5]

Appropriate books

Fenyman, Leighton and Sands *v. 3 Ch 1-3 of the Fenyman lectures on Physics.* Addison-Wesley 1970 (£87.99 paperback).

† S. Gasiorowicz *Quantum Physics.* Wiley 2003 (£34.95 hardback).

P.V. Landshoff, A.J.F. Metherell and W.G Rees *Essential Quantum Physics.* Cambridge University Press 1997 (£21.95 paperback).

† A.I.M. Rae *Quantum Mechanics.* Institute of Physics Publishing 2002 (£16.99 paperback).

L.I. Schiff *Quantum Mechanics.* McGraw Hill 1968 (£38.99 hardback).

ELECTROMAGNETISM **16 lectures, Lent term**

Introduction to Maxwell's equations

Electric and magnetic fields, charge, current. Maxwell's equations and the Lorentz force. Charge conservation. Integral form of Maxwell's equations and their interpretation. Scalar and vector potentials; gauge transformations. [3]

Electrostatics

Point charges and the inverse square law, line and surface charges, dipoles. Electrostatic energy. Gauss's law applied to spherically symmetric and cylindrically symmetric charge distributions. Plane parallel capacitor.

[3]

Steady currents

Ohm's law, flow of steady currents. Magnetic fields due to steady currents, simple examples treated by means of Ampère's equation. Vector potential due to a general current distribution, the Biot–Savart law. Magnetic dipoles. Lorentz force on steady current distributions and force between current-carrying wires. [4]

Electromagnetic induction

Faraday's law of induction for fixed and moving circuits; simple dynamo. [2]

Electromagnetic waves

Electromagnetic energy and Poynting vector. Plane electromagnetic waves in vacuum, polarisation. Reflection at a plane conducting surface. [4]

Appropriate books

W.N. Cottingham and D.A. Greenwood *Electricity and Magnetism.* Cambridge University Press 1991 (£17.95 paperback).

R. Feynman, R. Leighton and M. Sands *The Feynman Lectures on Physics, Vol 2.* Addison–Wesley 1970 (£87.99 paperback).

† P. Lorrain and D. Corson *Electromagnetism, Principles and Applications.* Freeman 1990 (£47.99 paperback).

J.R. Reitz, F.J. Milford and R.W. Christy *Foundations of Electromagnetic Theory.* Addison-Wesley 1993 (£46.99 hardback).

D.J. Griffiths *Introduction to Electrodynamics.* Prentice–Hall 1999 (£42.99 paperback).

INTRODUCTION TO SPECIAL RELATIVITY **8 lectures, Easter and Lent terms**

[Lecturers should use the signature convention $(+---)$*.]*

Space and time
The principle of relativity. Relativity and simultaneity. The invariant interval. Lorentz transformations in $(1+1)$-dimensional spacetime. Time dilation and muon decay. Length contraction. The Minkowski metric for $(1+1)$-dimensional spacetime. [4]

4–vectors
Lorentz transformations in $(3+1)$ dimensions. 4–vectors and Lorentz invariants. Proper time. 4–velocity and 4–momentum. Conservation of 4–momentum in radioactive decay. [4]

Appropriate books

G.F.R. Ellis and R.M. Williams *Flat and Curved Space-times.* Oxford University Press 2000 (£24.95 paperback).

† W. Rindler *Introduction to Special Relativity.* Oxford University Press 1991 (£19.99 paperback).

W. Rindler *Relativity: special, general and cosmological.* OUP 2001 (£24.95 paperback).

† E.F. Taylor and J.A. Wheeler *Spacetime Physics: introduction to special relativity.* Freeman 1992 (£29.99 paperback).

FLUID DYNAMICS **16 lectures, Michaelmas and Lent terms**

Kinematics

Continuum fields: density and velocity. Flow visualisation: particle paths, streamlines and dye streaklines. Material time derivative. Conservation of mass and the kinematic boundary condition at a moving boundary. Incompressibility and streamfunctions. [3]

Dynamics

Surface and volume forces; pressure in frictionless fluids. The Euler momentum equation. Applications of the momentum integral.

Bernoulli's theorem for steady flows with potential forces; applications.

Vorticity, vorticity equation, vortex line stretching, Kelvin's circulation theorem, irrotational flow remains irrotational. [4]

Potential flows

Velocity potential; Laplace's equation. Examples of solutions in spherical and cylindrical geometry by separation of variables.

Expression for pressure in time-dependent potential flows with potential forces; applications. Spherical bubbles; small oscillations, collapse of a void. Translating sphere and inertial reaction to acceleration, ∗effects of friction∗. Fluid kinetic energy for translating sphere.

Translating cylinder with circulation and lift force, ∗generation of circulation.∗ [6]

Interfacial flows

Governing equations and boundary conditions. Linear water waves: dispersion relation, deep and shallow water, particle paths, standing waves in a container. Rayleigh-Taylor instability.

River flows: over a bump, out of a lake over a broad weir, hydraulic jumps. [3]

Appropriate books

† D.J. Acheson *Elementary Fluid Dynamics.* Oxford University Press 1990 (£25.50 paperback).

G.K. Batchelor *An Introduction to Fluid Dynamics.* Cambridge University Press 2000 (£23.95 paperback).

E. Guyon, J.P. Hulin, L. Petit and C.D. Mitescu *Physical Hydrodynamics.* Oxford University Press 2001 (£32.50 paperback).

G.M. Homsey et. al. *Multi-Media Fluid Mechanics.* Cambridge University Press 2000 (CD-ROM for Windows or Macintosh, £14.95).

M.J. Lighthill *An Informal Introduction to Theoretical Fluid Mechanics.* Oxford University Press 1986 (out of print).

† A.R. Paterson *A First Course in Fluid Dynamics.* Cambridge University Press 1983 (£48.95 paperback).

M. van Dyke *An Album of Fluid Motion.* Parabolic Press 1982 (out of print).

NUMERICAL ANALYSIS **12 lectures, Easter term**

Linear equations and least squares calculations

LU triangular factorization of matrices. Relation to Gaussian elimination. Column pivoting. Factorizations of symmetric and band matrices. Iterative methods for linear equations. QR factorization of rectangular matrices by Gram–Schmidt, Givens and Householder techniques. Application to linear least squares calculations. [6]

Polynomial approximation

Interpolation by polynomials. Divided differences of functions and relations to derivatives. Orthogonal polynomials and their recurrence relations. Gaussian quadrature formulae. Peano Kernel theorem and applications. [6]

Appropriate books

† S.D. Conte and C. de Boor *Elementary Numerical Analysis: an algorithmic approach.* McGraw–Hill 1980 (out of print).

G.H. Golub and C. Van Loan *Matrix Computations.* Johns Hopkins University Press 1996 (out of print).

M.J.D. Powell *Approximation Theory and Methods.* Cambridge University Press 1981 (£25.95 paperback).

STATISTICS **16 lectures, Lent term**

Estimation

Review of distribution and density functions, parametric families, sufficiency, Rao–Blackwell theorem, factorization criterion, and examples: binomial, Poisson, gamma. Maximum likelihood estimation. Confidence intervals. Use of prior distributions and Bayesian inference. [5]

Hypothesis testing

Simple examples of hypothesis testing, null and alternative hypothesis, critical region, size, power, type I and type II errors, Neyman–Pearson lemma. Significance level of outcome. Uniformly most powerful tests. Likelihood ratio, and use of likelihood ratio to construct test statistics for composite hypotheses. Generalized likelihood-ratio test. Goodness-of-fit tests and contingency tables. [6]

Linear normal models

The χ^2, t and F distributions, joint distribution of sample mean and variance, Student's t-test, F-test for equality of two variances. One-way analysis of variance. [3]

Linear regression and least squares

Simple examples. $*$Use of software$*$. [2]

Appropriate books

D.A.Berry and B.W. Lindgren *Statistics, Theory and Methods.* Wadsworth 1996 (£36.00 paperback).

G. Casella and J.O. Berger *Statistical Inference.* Duxbury 2002 (£81.00 hardback).

M.H. DeGroot *Probability and Statistics.* Addison-Wesley 2002 (£42.99 paperback).

† J.A. Rice *Mathematical Statistics and Data Analysis.* Duxbury 1995 (out of print).

MARKOV CHAINS **24 lectures, Michaelmas term**

Discrete-time chains

Definition and basic properties, the transition matrix. Calculation of n-step transition probabilities. Communicating classes, closed classes, absorption, irreducibility. Calculation of hitting probabilities and mean hitting times; survival probability for birth and death chains. Stopping times and statement of the strong Markov property. [5]

Recurrence and transience; equivalence of transience and summability of n-step transition probabilities; equivalence of recurrence and certainty of return. Recurrence as a class property, relation with closed classes. Simple random walks in dimensions one, two and three. [3]

Invariant distributions, statement of existence and uniqueness up to constant multiples. Mean return time, positive recurrence; equivalence of positive recurrence and the existence of an invariant distribution. Convergence to equilibrium for irreducible, positive recurrent, aperiodic chains *and proof by coupling*. Long-run proportion of time spent in given state. [3]

Time reversal, detailed balance, reversibility; random walk on a graph. [1]

Appropriate books

G.R. Grimmett and D.R. Stirzaker *Probability and Random Processes.* OUP 2001 (£29.95 paperback).

J.R. Norris *Markov Chains.* Cambridge University Press 1997 (£20.95 paperback).

OPTIMISATION **12 lectures, Easter term**

Lagrangian methods

General formulation of constrained problems; the Lagrangian sufficiency theorem. Interpretation of Lagrange multipliers as shadow prices. Examples. [2]

Linear programming in the nondegenerate case

Convexity of feasible region; sufficiency of extreme points. Standardization of problems, slack variables, equivalence of extreme points and basic solutions. The primal simplex algorithm, artificial variables, the two-phase method. Practical use of the algorithm; the tableau. Examples. The dual linear problem, duality theorem in a standardized case, complementary slackness, dual variables and their interpretation as shadow prices. Relationship of the primal simplex algorithm to dual problem. Two person zero-sum games. [6]

Network problems

The Ford-Fulkerson algorithm and the max-flow min-cut theorems in the rational case. Network flows with costs, the transportation algorithm, relationship of dual variables with nodes. Examples. Conditions for optimality in more general networks; ∗the simplex-on-a-graph algorithm∗. [3]

Practice and applications

∗Efficiency of algorithms∗. The formulation of simple practical and combinatorial problems as linear programming or network problems. [1]

Appropriate books

† M.S. Bazaraa, J.J. Jarvis and H.D. Sherali *Linear Programming and Network Flows.* Wiley 1990 (£80.95 hardback).

† D. Luenberger *Linear and Nonlinear Programming.* Addison–Wesley 1984 (out of print).

R.J. Vanderbei *Linear programming: foundations and extensions.* Kluwer 2001 (£61.50 hardback).

COMPUTATIONAL PROJECTS **6 lectures (Easter term) plus project work**

GENERAL INFORMATION

This course is examined in a different way from other Mathematical Tripos courses. Each project requires practical work on a computer. The emphasis is on the mathematical nature of the project rather than on programming or numerical techniques. Credit for examination purposes is obtained by the submission of written reports and no questions are set in the written papers. There are two core projects which everybody is expected to attempt and a choice of two additional projects from four available. Marks awarded are added to the totals for written papers. The maximum credit available is approximately two thirds of that for a 24 lecture course. The examination credit is not related to the number of lectures, but to the projects which are submitted.

Suitable equipment for the project work is available in the Maths Public Workstation Facility (PWF) situated in the basement of Pavilion G at the Centre for Mathematical Sciences where a demonstrator will be present (at specified times) to give advice and assistance. Appropriate software is provided by the Faculty for this course which may be accessed from the Maths PWF. Similar equipment and software is available at other PWFs at various University sites and in most colleges.

Subject to standard Computing Service rules, the Maths PWF may be used by anyone who is a candidate for the Mathematical Tripos. Excepting unforeseeable circumstances such as hardware failure or power cuts, it is normally open from 8.30 a.m. to 5.30 p.m. every weekday in Full Term (8.30 a.m. to
1.00 p.m. on Saturdays during Michaelmas and Lent terms only). From time to time it may be necessary to reserve the room for other purposes, and the precise opening hours are subject to variations, which will be announced as much in advance as possible.

The software which the Faculty has written and provided specifically for this course is available for use on students' own machines (IBM PC-compatible) but the Faculty cannot undertake to provide copies of any other copyright proprietary software. Students intending to use their own personal equipment may wish to purchase copies of such software for themselves.

Further information on the Computational Projects is available on the Faculty website at `http://www.maths.cam.ac.uk/catam/`

PROJECT WORK

The Faculty publishes a projects booklet by the end of July preceding the Part IB year (i.e., shortly after the end of Term 3). This contains the projects which may be attempted and further details about course administration. Once the booklet is available, the projects may be done at any time up to the submission deadlines, which are near the start of the Full Lent Term in the IB year (i.e. in Term 5) for the two core projects and near the start of the Full Easter Term in the IB year (i.e. in Term 6) for the two additional projects as specified in the projects booklet.

LECTURES

The lectures are given in the Easter Full Term of the Part IA year and are accompanied by introductory practical classes. Students register for these classes: arrangements will be announced during the lectures.

Introduction to course
Examples of the use of computers in mathematics. Overview of course, including administration, marks and computing facilities. Introduction to the projects.

Using Windows and CCATSL elements. Introduction to programming in C (editor, compiler and graphics)
Structure of a program, and programming techniques; manipulation of arrays, iteration and recursion. Basic C commands. Complexity of an algorithm.

Introduction to software libraries and packages
How to call mathematical and graphical software libraries from C with illustrations, for example, numerical solution of ordinary differential equations.

Example project
The key points of how to complete a project should be covered by illustration.

Appropriate books

† Faculty of Mathematics, University of Cambridge *Learning to use C and the CATAM software library.* Available online or from CMS Reception (£2.00).

† Faculty of Mathematics, University of Cambridge *The CATAM Software Library.* Available from online or from CMS Reception (£4.00).

† S.D. Conte and C. de Boor *Elementary Numerical Analysis.* McGraw–Hill 1980 (out of print).

C.F. Gerald and P.O. Wheatley *Applied Numerical Analysis.* 6th edition, Addison–Wesley 1999 (£67.99 hardback). 7th ed. entitled 'Numerical Analysis' due for publication September 2003

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