THE MATHEMATICAL TRIPOS 2005-2006

CONTENTS

This booklet contains the schedule, or syllabus specification, for each course of the undergraduate Tripos together with information about the examinations. It is updated every year. Suggestions and corrections should be e-mailed to faculty@maths.cam.ac.uk.

SCHEDULES

Syllabus

The schedule for each course is a list of topics that define the course. The schedule is agreed by the Faculty Board.

Some schedules contain topics which are 'starred' (listed between asterisks). The set of unstarred topics is minimal for lecturing and maximal for examining; which means that all the topics must be covered by the lecturer and that examiners must not set questions on material not listed in the schedule. Lecturers should also cover any topics starred in the schedules, but questions will not be set on them in examinations.

The numbers which appear in brackets at the end of subsections or paragraphs in these schedules indicate the approximate number of lectures likely to be devoted to the subsection or paragraph concerned. Lecturers decide upon the amount of time they think appropriate to spend on each topic, and also on the order in which they present to topics. In Parts IA and IB, there is not much flexibility, because some topics have to be introduced by a certain time in order to be used in other courses.

Recommended books

A list of books is given after each schedule. Books marked with † are particularly well suited to the course. Some of the books are out of print; these are retained on the list because they should be available in college libraries (as should all the books on the list) and may be found in second-hand bookshops. There may well be many other suitable books not listed; it is usually worth browsing.

Most books on the list have a price attached: this is based on the most up to date information available at the time of production of the schedule, but it may not be accurate.

EXAMINATIONS

Overview

There are three examinations for the undergraduate Mathematical Tripos, Parts IA, IB and II, which are normally taken in consecutive years. On this page, details of the examination that are common to all three Parts of the undergraduate Tripos are explained. Specific details are given later in this booklet in the *General Arrangements* sections for the Part of the Tripos.

The form of each examination (number of papers, numbers of questions on each lecture course, distribution of questions in the papers and in the sections of each paper, number of questions which may be attempted) is determined by the Faculty Board. The main structure has to be agreed by University committees and is published as a Regulation in the Statutes and Ordinances of the University of Cambridge. Significant changes are announced in the *Reporter* as a Form and Conduct notice. The actual questions, marking schemes and precise borderlines are determined by the examiners.

The examiners for each Part of the Tripos are normally teaching staff in the two departments together with one or more external examiners from other universities and are appointed by the General Board of the University. The list of examiners is published in the Reporter.

Form of the examination

Each examination consists of four written papers and candidates take all four. For Parts IB and II, candidates may in addition submit Computational Projects. Each written paper has two sections: Section I contains questions that are intended to be accessible to any student who has studied the material conscientiously. They should not contain any significant 'problem' element. Section II questions are intended to be more challenging

Calculators are not allowed in any paper of Parts IB and II of the Tripos, nor in Papers 1–4 of Part IA. Examination questions will be set in such a way as not to require the use of calculators. The rules for the use of calculators in the Computer Science and Physics papers of Options (b) and (c) of Part IA are set out in the regulations for the relevant Tripos.

Formula booklets are not permitted, but candidates will not be required to quote elaborate formulae from memory.

Data Protection Act

To meet the University's obligations under the Data Protection Act 1998, the Faculty deals with data relating to individuals and their examination marks as follows:

- Marks for individual questions and Computational Projects are released routinely after the examinations.
- Scripts are kept, in line with the University policy, for four months following the examinations (in case of appeals) and are then destroyed.
- Neither the Data Protection Act nor the Freedom of Information Act entitle candidates to have access to their scripts. However, data appearing on individual examination scripts are available on application to the University Data Protection Officer and on payment of a fee. Such data would consist of little more than ticks, crosses, underlines, and mark subtotals and totals.

Marking conventions

Section I questions are marked out of 10 and Section II questions are marked out of 20. In addition to a numerical mark, extra credit in the form of a quality mark may be awarded for each question depending on the completeness and quality of each answer. For a Section I question, a *beta* quality mark is awarded for a mark of 8 or more. For a Section II question, an *alpha* quality mark is awarded for a mark of 15 or more, and a *beta* quality mark is awarded for a mark between 10 and 14, inclusive.

A merit mark, call it M, is calculated for each candidate according to the following formula:

$$M = \begin{cases} m + 10\alpha + 3\beta - 24 & \text{if } \alpha \geq 8, \\ m + 7\alpha + 3\beta & \text{if } \alpha \leq 8. \end{cases}$$

It is used only as a convenience by examiners to produce an initial list in an order which, experience shows, corresponds more closely to the final order than that produced on the basis of marks alone. The class borderlines are not determined by merit mark; rather, the merit mark formula is determined by the class borderlines of previous years.

The Faculty Board recommends that no distinction should be made between marks obtained on the Computational Projects courses and marks obtained on the written papers, and no requirement is placed on candidates to produce answers on a range of mathematical material beyond that imposed by the distribution of questions on the papers.

On some papers, there are restrictions on the number of questions that should be attempted, indicated by a rubric of the form '*You should attempt at most N questions in Section I*'. The Faculty policy is that examiners mark all attempts, even if the number of these exceeds that specified, and assess the candidate on the best attempts consistent with the restriction. This policy is intended to deal with candidates who accidently violate the rubric: it is clearly not in candidates' best interests to violate the rubric knowingly.

Examinations are 'single-marked', but safety checks are made on all scripts to ensure that all work is marked and that all marks are correctly added and transcribed. Scripts are identified only by candidate number until the final class-lists have been drawn up. In drawing up the class list, examiners make decisions based only on the work they see: no account is taken of the candidates' personal situation or of supervision reports. Candidates for whom a warning letter has been received may be removed from the class list pending an appeal. All appeals must be made through official channels (via a college tutor); examiners should not be approached either by candidates or their directors of studies as this might jeopardise any formal appeal.

Classification

As a result of each examination, each candidate is placed in one of the following categories: 1, 2.1, 2.2, 3, ordinary, fail or 'other'. 'Other' here includes, for example, candidates who were ill for part of the examination. A candidate who is placed in the ordinary or fail categories cannot continue unless there are extenuating circumstances on which to base an appeal to the Council of the University.

Quality marks as well as numerical marks are taken into account by the examiners in deciding the class borderlines. The Faculty Board has recommended that the number of alphas should be of particular importance at the first/second borderline but that at the lower borderlines alphas, and betas, and total mark should each (individually or together) be regarded as indicators of quality. At the third/ordinary and ordinary/fail borderlines, individual considerations are always paramount. Very careful scrutiny is given to candidates near any borderlines and other factors besides marks and quality marks may be taken into account.

The Faculty Board recommends approximate percentages of candidates for each class (30% firsts, 40-45% upper seconds, 20-25% lower seconds, and not more than 6% thirds and below).

The Faculty Board also recommends to examiners the following criteria for deciding the different classes.

The First Class

Candidates placed in the first class will have demonstrated a good command and secure understanding of examinable material. They will have presented standard arguments accurately, showed skill in applying their knowledge, and generally will have produced substantially correct solutions to a significant number of more challenging questions.

The Upper Second Class

Candidates placed in the upper second class will have demonstrated good knowledge and understanding of examinable material. They will have presented standard arguments accurately and will have shown some ability to apply their knowledge to solve problems. A fair number of their answers to both straightforward and more challenging questions will have been substantially correct.

The Lower Second Class

Candidates placed in the lower second class will have demonstrated knowledge but sometimes imperfect understanding of examinable material. They will have been aware of relevant mathematical issues, but their presentation of standard arguments will sometimes have been fragmentary or imperfect. They will have produced substantially correct solutions to some straightforward questions, but will have had limited success at tackling more challenging problems.

The Third Class

Candidates placed in the third class will have demonstrated some knowledge but little understanding of the examinable material. They will have made reasonable attempts at a small number of questions, but will have lacked the skills to complete many of them.

Ordinary

Candidates granted an allowance towards an Ordinary Degree will have demonstrated knowledge of a small amount of examinable material by making reasonable attempts at some straightforward questions.

MISCELLANEOUS MATTERS

Numbers of supervisions

Directors of Studies will arrange supervisions for each course as they think appropriate. Lecturers will hand out examples sheets which supervisors may use if they wish. According to Faculty Board guidelines, the number of examples sheets for 24-lecture, 16-lecture and 12-lecture courses should be 4, 3 and 2 respectively.

Feedback

Constructive feedback of all sorts and from all sources is welcomed by everyone concerned in providing courses for the Mathematical Tripos.

There are many different feedback routes. Each lecturer hands out a questionnaire towards the end of the course and students are sent a combined e-mail questionnaire at the end of each year. These questionnaires are particularly important in shaping the future of the Tripos and the Faculty Board urge all students to respond. To give feedback during the course of the term, students can e-mail the Faculty Board and Teaching Committee students representatives and anyone can e-mail the anonymous rapid response Faculty 'hotline' `hotline@maths.cam.ac.uk`. Feedback can be sent directly to the Teaching Committee via the Faculty Office. Feedback on college-provided teaching can be given to Directors of Studies or Tutors at any time.

Student representatives

There are three student representatives on the Faculty Board, and two on each of the the Teaching Committee and the Curriculum Committee. They are normally elected (in the case of the Faculty Board representatives) or appointed in November of each year. Their role is to advise the committees on the student point of view, to collect opinion from and liaise with the student body. They operate a website: `http://www.damtp.cam.ac.uk/user/studrep` and their email address is `student.reps@damtp.cam.ac.uk`.

Aims and objectives

The **aims** of the Faculty for Parts IA, IB and II of the Mathematical Tripos are:

- to provide a challenging course in mathematics and its applications for a range of students that includes some of the best in the country;
- to provide a course that is suitable both for students aiming to pursue research and for students going into other careers;
- to provide an integrated system of teaching which can be tailored to the needs of individual students;
- to develop in students the capacity for learning and for clear logical thinking;
- to continue to attract and select students of outstanding quality;
- to produce the high calibre graduates in mathematics sought by employers in universities, the professions and the public services.
- to provide an intellectually stimulating environment in which students have the opportunity to develop their skills and enthusiasms to their full potential;
- to maintain the position of Cambridge as a leading centre, nationally and internationally, for teaching and research in mathematics.

The **objectives** of Parts IA, IB and II of the Mathematical Tripos are as follows:

After completing Part IA, students should have:

- made the transition in learning style and pace from school mathematics to university mathematics;
- been introduced to basic concepts in higher mathematics and their applications, including (i) the notions of proof, rigour and axiomatic development, (ii) the generalisation of familiar mathematics to unfamiliar contexts, (iii) the application of mathematics to problems outside mathematics;
- laid the foundations, in terms of knowledge and understanding, of tools, facts and techniques, to proceed to Part IB.

After completing Part IB, students should have:

- covered material from a range of pure mathematics, statistics and operations research, applied mathematics, theoretical physics and computational mathematics, and studied some of this material in depth;
- acquired a sufficiently broad and deep mathematical knowledge and understanding to enable them both to make an informed choice of courses in Part II and also to study these courses.

After completing Part II, students should have:

- developed the capacity for (i) solving both abstract and concrete problems, (ii) presenting a concise and logical argument, and (iii) (in most cases) using standard software to tackle mathematical problems;
- studied advanced material in the mathematical sciences, some of it in depth.

Part IA

GENERAL ARRANGEMENTS

Structure of Part IA

There are three options:

(*a*) Pure and Applied Mathematics;
(*b*) Mathematics with Computer Science;
(*c*) Mathematics with Physics.

Option (*a*) is intended primarily for students who expect to continue to Part IB of the Mathematical Tripos, while Options (*b*) and (*c*) are intended primarily for those who are undecided about whether they will continue to Part IB of the Mathematical Tripos or change to Part IB of the Computer Science Tripos or Part IB of the Natural Sciences Tripos (Physics option).

For Option (*b*), one of the lecture courses (Dynamics) is replaced by courses from the Computer Science Tripos. For Option (*c*), two of the lecture courses (Numbers and Sets and Dynamics) are replaced by courses from the Natural Sciences Tripos. Students wishing to examine the schedules for these courses should consult the documentation of the appropriate faculty, for example on http://www.cl.cam.ac.uk/UoCCL/teaching/ and http://www.phy.cam.ac.uk/teaching/.

Examinations

Arrangements common to all examinations of the undergraduate Mathematical Tripos are given on pages 1 and 2 of this booklet.

All candidates for Part IA of the Mathematical Tripos take four papers, as follows.
Candidates taking Option (*a*) (Pure and Applied Mathematics) will take Papers 1, 2, 3 and 4 of the Mathematical Tripos (Part IA).
Candidates taking Option (*b*), Mathematics with Computer Science, take Papers 1, 2 and 3 of the Mathematical Tripos (Part IA) and a paper from the Computer Science Tripos, which examines material from Computer Science and Numbers and Sets; they must also submit a portfolio of assessed laboratory work.
Candidates taking Option (*c*), Mathematics with Physics, take Papers 1, 2 and 3 of the Mathematical Tripos (Part IA) and the Physics paper of the Natural Sciences Tripos (Part IA); they must also submit practical notebooks.

Papers 1, 2, 3 and 4 from the Mathematical Tripos (Part IA) are divided into two Sections. There are four questions in Section I and eight questions in Section II. Candidates may attempt all the questions in Section I and should attempt at most five questions from Section II, of which no more than three may be on the same course.

For Mathematics with Computer Science or Mathematics with Physics candidates, the marks and quality marks for the Computer Science or Physics paper are scaled to bring them in line with Paper 4.

Each section of each of Paper 1—4 is divided equally between two courses as follows:

Paper 1:	Algebra and Geometry, Analysis I
Paper 2:	Differential Equations, Probability
Paper 3:	Algebra and Geometry, Vector Calculus
Paper 4:	Numbers and Sets, Dynamics.

The following table shows the approximate mark, number of alphas, number of betas and merit mark of a typical candidate placed near the bottom of each class last year. This may be taken as a rough guide for the current year, but it is important to realise that borderlines may go up or down. At the lowest borderlines, individual consideration of candidates is always paramount, so there are no typical candidates; *very few* candidates fail to achieve at least a third class.

2005	Borderline	Borderline candidate
	1/2(i)	354,11,11; 473
	2(i)/2(ii)	256,7,11; 338
	2(ii)/3	199,4,9; 254

ALGEBRA AND GEOMETRY — 48 lectures, Michaelmas term

Review of complex numbers, modulus, argument and de Moivre's theorem. Informal treatment of complex logarithm, *n-th* roots and complex powers. Equations of circles and straight lines. Examples of Möbius transformations. [3]

Vectors in R^3. Elementary algebra of scalars and vectors. Scalar and vector products, including triple products. Geometrical interpretation. Cartesian coordinates; plane, spherical and cylindrical polar coordinates. Suffix notation: including summation convention and δ_{ij}, ϵ_{ijk}. [5]

Vector equations. Lines, planes, spheres, cones and conic sections. Maps: isometries and inversions. [2]

Introduction to $\mathbb{R}^n$, scalar product, Cauchy–Schwarz inequality and distance. Subspaces, brief introduction to spanning sets and dimension. [4]

Linear maps from $\mathbb{R}^m$ to $\mathbb{R}^n$ with emphasis on $m, n \leqslant 3$. Examples of geometrical actions (reflections, dilations, shears, rotations). Composition of linear maps. Bases, representation of linear maps by matrices, the algebra of matrices. [5]

Determinants, non-singular matrices and inverses. Solution and geometric interpretation of simultaneous linear equations (3 equations in 3 unknowns). Gaussian Elimination. [3]

Discussion of $\mathbb{C}^n$, linear maps and matrices. Eigenvalues, the fundamental theorem of algebra (statement only), and its implication for the existence of eigenvalues. Eigenvectors, geometric significance as invariant lines. [3]

Discussion of diagonalization, examples of matrices that cannot be diagonalized. A real 3×3 orthogonal matrix has a real eigenvalue. Real symmetric matrices, proof that eigenvalues are real, and that distinct eigenvalues give orthogonal basis of eigenvectors. Brief discussion of quadratic forms, conics and their classification. Canonical forms for 2×2 matrices; discussion of relation between eigenvalues of a matrix and fixed points of the corresponding Möbius map. [5]

Axioms for groups; subgroups and group actions. Orbits, stabilizers, cosets and conjugate subgroups. Orbit-stabilizer theorem. Lagrange's theorem. Examples from geometry, including the Euclidean groups, symmetry groups of regular polygons, cube and tetrahedron. The Möbius group; cross-ratios, preservation of circles, informal treatment of the point at infinity. [11]

Isomorphisms and homomorphisms of abstract groups, the kernel of a homomorphism. Examples. Introduction to normal subgroups, quotient groups and the isomorphism theorem. Permutations, cycles and transpositions. The sign of a permutation. [5]

Examples (only) of matrix groups; for example, the general and special linear groups, the orthogonal and special orthogonal groups, unitary groups, the Lorentz groups, quaternions and Pauli spin matrices. [2]

Appropriate books

M.A. Armstrong *Groups and Symmetry.* Springer–Verlag 1988 (£33.00 hardback).
† Alan F Beardon *Algebra and Geometry.* CUP 2005 (£21.99 paperback, £48 hardback).
D.M. Bloom *Linear Algebra and Geometry.* Cambridge University Press 1979 (out of print).
D.E. Bourne and P.C. Kendall *Vector Analysis and Cartesian Tensors.* Nelson Thornes 1992 (£30.75 paperback).
R.P. Burn *Groups, a Path to Geometry.* Cambridge University Press 1987 (£20.95 paperback).
J.A. Green *Sets and Groups: a first course in Algebra.* Chapman and Hall/CRC 1988 (£38.99 paperback).
E. Sernesi *Linear Algebra: A Geometric Approach.* CRC Press 1993 (£38.99 paperback).
D. Smart *Linear Algebra and Geometry.* Cambridge University Press 1988 (out of print).

NUMBERS AND SETS — 24 Lectures, Michaelmas term

[Note that this course is omitted from Option (c) of Part IA.]

Introduction to numbers systems and logic

Overview of the natural numbers, integers, real numbers, rational and irrational numbers, algebraic and transcendental numbers. Brief discussion of complex numbers; statement of the Fundamental Theorem of Algebra.

Ideas of axiomatic systems and proof within mathematics; the need for proof; the role of counter-examples in mathematics. Elementary logic; implication and negation; examples of negation of compound statements. Proof by contradiction. [2]

Sets, relations and functions

Union, intersection and equality of sets. Indicator (characteristic) functions; their use in establishing set identities. Functions; injections, surjections and bijections. Relations, and equivalence relations. Counting the combinations or permutations of a set. The Inclusion-Exclusion Principle. [4]

The integers

The natural numbers: the well-ordering principle, and its equivalence with Mathematical Induction. Examples, including the Binomial Theorem. [2]

Elementary number theory

Prime numbers: existence and uniqueness of prime factorisation into primes; highest common factors and least common multiples. Euclid's proof of the infinity of primes. Euclid's algorithm. Solution in integers of ax+by = c.

Modular arithmetic (congruences). Units modulo n. Chinese Remainder Theorem. Wilson's Theorem; the Fermat-Euler Theorem. Public key cryptography and the RSA algorithm. [10]

The real numbers

Irrational numbers, including $\sqrt{2}$ and e. Decimal expansions. Construction of a transcendental number. [2]

Countability and uncountability

Definitions of finite, infinite, countable and uncountable sets. A countable union of countable sets is countable. Uncountability of $\mathbb{R}$. Non-existence of a bijection from a set to its power set. Indirect proof of existence of transcendental numbers. [4]

Appropriate books

R.B.J.T. Allenby *Numbers and Proofs.* Butterworth-Heinemann 1997 (£19.50 paperback).
R.P. Burn *Numbers and Functions: steps into analysis.* Cambridge University Press 2000 (£21.95 paperback).
H. Davenport *The Higher Arithmetic.* Cambridge University Press 1999 (£19.95 paperback).
A.G. Hamilton *Numbers, sets and axioms: the apparatus of mathematics.* Cambridge University Press 1983 (£20.95 paperback).
C. Schumacher *Chapter Zero: Fundamental Notions of Abstract Mathematics.* Addison-Wesley 2001 (£42.95 hardback).
I. Stewart and D. Tall *The Foundations of Mathematics.* Oxford University Press 1977 (£22.50 paperback).

DIFFERENTIAL EQUATIONS 24 lectures, Michaelmas term

Basic calculus
Informal treatment of differentiation as a limit, the chain rule, Leibnitz's rule, Taylor series; integration as an area, fundamental theorem of calculus, integration by substitution and parts. [2]

First-order linear differential equations
Equations with constant coefficients: exponential growth, comparison with discrete equations, series solution; modelling examples including radioactive decay.

Equations with non-constant coefficients: solution by integrating factor, series solution, comparison with discrete equations. [3]

Higher-order linear differential equations
Complementary function and particular integral, linear independence, Wronskian (for second-order equations). Equations with constant coefficients and examples including radioactive sequences, comparison in simple cases with difference equations, reduction of order, resonance, transients, damping. Homogeneous equations. Response to step and impulse function inputs. Coupled first order systems: equivalence to single higher-order equations; solution by matrix methods. Series solutions including statement only of the need for the logarithmic solution. [8]

Partial differentiation
Informal treatment of partial derivatives, geometrical interpretation, statement (only) of symmetry of mixed partial derivatives, chain rule. Informal treatment of differentials. Simple examples of first- and second-order partial differential equations, solution of the wave equation in the form $f(x+ct)+g(x-ct)$. Statement of Taylor series for functions on $\mathbb{R}^n$. Local extrema of real functions, classification using the Hessian matrix. Differentiation of an integral with respect to a parameter. [6]

Nonlinear equations first-order equations
Separable equations. Exact equations. Families of solutions, isoclines, the idea of a flow and connection with vector fields. Equilibrium solutions, stability by perturbation; examples, including logistic equation and chemical kinetics. Discrete equations: equilibrium solutions, stability; examples including the logistic equation. [5]

Appropriate books

W.E. Boyce and R.C. DiPrima *Elementary Differential Equations and Boundary-Value Problems.* Wiley 7th edition 2001 (£34.95 hardback). 8th ed. due for publication in May 2004
D.N. Burghes and M.S. Borrie *Modelling with Differential Equations.* Ellis Horwood 1981 (out of print).
W. Cox *Ordinary Differential Equations.* Butterworth-Heinemann 1996 (£14.99 paperback).
F. Diacu *An introduction to Differential Equations: Order and Chaos.* Freeman 2000 (£38.99 hardback).
N. Finizio and G. Ladas *Ordinary Differential Equations with Modern Applications.* Wadsworth 1989 (out of print).
D. Lomen and D. Lovelock *Differential Equations: Graphics-Models-Data.* Wiley 1999 (£80.95 hardback).
R.E. O'Malley *Thinking about Ordinary Differential Equations.* Cambridge University Press 1997 (£19.95 paperback).
D.G. Zill and M.R. Cullen *Differential Equations with Boundary Value Problems.* Brooks/Cole 2001 (£37.00 hardback).

ANALYSIS I 24 lectures, Lent term

Limits and convergence
Limit of sequences in $\mathbb{R}$ and $\mathbb{C}$. Sums, products and quotients. Axiom that bounded monotonic sequences in $\mathbb{R}$ converge. Convergent series, linear combinations of series. Absolute convergence; absolute convergence implies convergence. Comparison and ratio tests, alternating series test. [5]

Least upper bounds
Least upper bounds and greatest lower bounds; simple examples. Least upper bound axiom and its equivalence with convergence of bounded monotonic sequences. Applications. [3]

Continuity
Continuity of real- and complex-valued functions defined on subsets of $\mathbb{R}$ and $\mathbb{C}$. The intermediate value theorem. The Bolzano-Weierstrass theorem. A continuous function on a closed bounded interval is bounded and attains its bounds. [3]

Differentiability
Differentiability of functions from $\mathbb{R}$ to $\mathbb{R}$. Derivative of sums and products. The chain rule. Derivative of the inverse function. Rolle's theorem; the mean value theorem. One-dimensional version of the inverse function theorem. Taylor's theorem from $\mathbb{R}$ to $\mathbb{R}$; Lagrange's form of the remainder. Complex differentiation. Taylor's theorem from $\mathbb{C}$ to $\mathbb{C}$ (statement only). [4]

Power series
Complex power series and radius of convergence. Exponential, trigonometric and hyperbolic functions, and relations between them. *Direct proof of the differentiability of a power series within its circle of convergence*. [4]

Integration
Definition and basic properties of the Riemann integral. A non-integrable function. Integrability of monotonic functions. Integrability of piecewise-continuous functions (statement only). The fundamental theorem of calculus. Differentiation of indefinite integrals. Integration by parts. The integral form of the remainder in Taylor's theorem. Improper integrals. [5]

Appropriate books

H. Anton *Calculus.* Wiley 2002 (£41.71 paperback with CD-ROM).
T.M. Apostol *Calculus.* Wiley 1967-69 (£181.00 hardback).
J.C. Burkill *A First Course in Mathematical Analysis.* Cambridge University Press 1978 (£18.95 paperback).
J.B. Reade *Introduction to Mathematical Analysis.* Oxford University Press (out of print).
M. Spivak *Calculus.* Addison–Wesley/Benjamin–Cummings 1967 (out of print).

PROBABILITY

24 lectures, Lent term

Basic concepts
Classical probability, equally likely outcomes. Combinatorial analysis, permutations and combinations. Stirling's formula (asymptotics for $\log n!$ proved). [3]

Axiomatic approach
Axioms (countable case). Probability spaces. Addition theorem, inclusion-exclusion formula. Boole and Bonferroni inequalities. Independence. Binomial, Poisson and geometric distributions. Relation between Poisson and binomial distributions. Conditional probability, Bayes's formula. Examples, including Simpson's paradox. [5]

Discrete random variables
Expectation. Functions of a random variable, indicator function, variance, standard deviation. Covariance, independence of random variables. Generating functions: sums of independent random variables, random sum formula, moments.

Conditional expectation. Random walks: gambler's ruin, recurrence relations. Difference equations and their solution. Mean time to absorption. Branching processes: generating functions and extinction probability. Combinatorial applications of generating functions. [7]

Continuous random variables
Distributions and density functions. Expectations; expectation of a function of a random variable. Uniform, normal and exponential random variables. Memoryless property of exponential distribution.

Joint distributions: transformation of random variables, examples. Simulation: generating continuous random variables, independent normal random variables. Geometrical probability: Bertrand's paradox, Buffon's needle. Correlation coefficient, bivariate normal random variables. [6]

Inequalities and limits
Markov's inequality, Chebyshev's inequality. Weak law of large numbers. Convexity: Jensen's inequality, AM/GM inequality.

Moment generating functions. Statement of central limit theorem and sketch of proof. Examples, including sampling. [3]

Appropriate books

W. Feller *An Introduction to Probability Theory and its Applications, Vol. I.* Wiley 1968 (£73.95 hardback).

† G. Grimmett and D. Welsh *Probability: An Introduction.* Oxford University Press 1986 (£23.95 paperback).

† S. Ross *A First Course in Probability.* Wiley 2002 (£39.99 paperback).

D.R. Stirzaker *Elementary Probability.* Cambridge University Press 1994/2003 (£19.95 paperback).

VECTOR CALCULUS

24 lectures, Lent term

Curves in $\mathbb{R}^3$
Parameterised curves and arc length, tangents and normals to curves in $\mathbb{R}^3$, the radius of curvature. [1]

Integration in $\mathbb{R}^2$ and $\mathbb{R}^3$
Line integrals. Surface and volume integrals: definitions, examples using Cartesian, cylindrical and spherical coordinates; change of variables. [4]

Vector operators
Directional derivatives. The gradient of a real-valued function: definition; interpretation as normal to level surfaces; examples including the use of cylindrical, spherical *and general orthogonal curvilinear* coordinates; conservative fields. Divergence, curl and ∇^2 in Cartesian coordinates, examples; formulae for these operators (statement only) in cylindrical, spherical *and general orthogonal curvilinear* coordinates. Solenoidal fields and irrotational fields. Vector derivative identities. [5]

Integration theorems
Divergence theorem, Green's theorem, Stokes's theorem, Green's second theorem: statements; informal proofs; examples; application to fluid dynamics and electromagnetism. [5]

Laplace's equation
Laplace's equation in $\mathbb{R}^2$ and $\mathbb{R}^3$: uniqueness theorem and maximum principle. Solution of Poisson's equation by Gauss's method (for spherical and cylindrical symmetry) and as an integral. [4]

Cartesian tensors in $\mathbb{R}^3$
Tensor transformation laws, addition, multiplication, contraction, with emphasis on tensors of second rank. Isotropic second and third rank tensors. Symmetric and antisymmetric tensors. Revision of principal axes and diagonalization. Quotient theorem. Examples including inertia and conductivity. [5]

Appropriate books

H. Anton *Calculus.* Wiley Student Edition 2000 (£33.95 hardback).

T.M. Apostol *Calculus.* Wiley Student Edition 1975 (Vol. II £37.95 hardback).

M.L. Boas *Mathematical Methods in the Physical Sciences.* Wiley 1983 (£32.50 paperback).

† D.E. Bourne and P.C. Kendall *Vector Analysis and Cartesian Tensors.* 3rd edition, Nelson Thornes 1999 (£29.99 paperback).

E. Kreyszig *Advanced Engineering Mathematics.* Wiley International Edition 1999 (£30.95 paperback, £97.50 hardback).

J.E. Marsden and A.J.Tromba *Vector Calculus.* Freeman 1996 (£35.99 hardback).

P.C. Matthews *Vector Calculus.* SUMS (Springer Undergraduate Mathematics Series) 1998 (£18.00 paperback).

† K. F. Riley, M.P. Hobson, and S.J. Bence *Mathematical Methods for Physics and Engineering.* Cambridge University Press 2002 (£27.95 paperback, £75.00 hardback).

H.M. Schey *Div, grad, curl and all that: an informal text on vector calculus.* Norton 1996 (£16.99 paperback).

M.R. Spiegel *Schaum's outline of Vector Analysis.* McGraw Hill 1974 (£16.99 paperback).

DYNAMICS 24 lectures, Lent term

[Note that this course is omitted from Options (b) and (c) of Part IA.]

Basic concepts
Motion, reference frames and units; acceleration; Newton's laws, mass and force; Galilean transformation; forces including gravity, electrostatic, Lorentz, elastic, friction.

Dimensional analysis, scaling and use of dimensionless parameters. [3]

Dynamical examples
Circular orbits, electron in a uniform electromagnetic field, simple harmonic motion, projectiles with quadratic friction, varying mass, the rocket equation, constrained motion. [3]

Energy
Work, kinetic energy, potential energy in one dimension and in three dimensions for conservative forces. Motion and the shape of the potential energy function; stability of equilibria, escape velocity.

Small amplitude oscillations in one dimension. [3]

Collisions
Impulse and collisions (elastic and inelastic): centre of mass, energy, coefficient of restitution. [1]

Rotation
Acceleration in plane polar coordinates. Angular velocity, angular momentum in three dimensions. Rotating frames; apparent gravity, centrifugal and Coriolis accelerations. Foucault pendulum. [3]

Orbits
Gravitational potential for a sphere; escape velocity; shape of the orbit for $1/r^2$ forces; orbital equations in terms of $u(\theta)$; period; Kepler's laws; stability of circular orbits. Rutherford scattering, impact parameter and cross-section. [4]

Systems of particles
Momentum, kinetic energy and angular momentum about centre of mass; reduced mass in 2-body problem. [1]

Rotation of rigid bodies
Moments of inertia, parallel axis theorem. Simple examples of rotation and translation with a constant axis. [3]

Phase-plane techniques
Classification of equilibria; conservative system; damped systems: ∗limit cycles∗. Forced pendulum, ∗hysteresis, chaos∗. [3]

Appropriate books

† V.D. Barger and M.G. Olsson *Classical Mechanics: a modern perspective.* McGraw-Hill 1995 (out of print).
D.N. Burghes and A.M. Downs *Modern Introduction to Classical Mechanics and Control.* Ellis Horwood 1975 (out of print).
C.D. Collinson *Introductory Mechanics.* Arnold 1980 (out of print).
† A.P. French and M.G. Ebison *Introduction to Classical Mechanics.* Kluwer 1986 (£33.25 paperback).
M. A. Lunn *A First Course in Mechanics.* Oxford University Press 1991 (£17.50 paperback).

PHYSICS (non-examinable) 12 lectures, Michaelmas term

Idea of mass and force, Newton's law $F = ma$. Motion of projectiles. Simple pendulum. Circular motion, angular velocity. Examples of simple mechanics problems. Hooke's law.

Boyle's law, pressure, temperature, ideal gases. Heat as a form of energy.

Electric currents, voltage, Ohm's law. Simple circuits, resistances in parallel and series. Capacitors and inductances in circuits. Electric and magnetic fields.

Elementary ideas in optics, Snell's law, Fermat's principle. Light as a wave motion, interference, polarization. Infra-red and ultra-violet radiation, radio waves, the electromagnetic theory of light and the electromagnetic spectrum.

Structure of atoms, very simple treatment of hydrogen atom using quantisation of angular momentum. Bohr radius as characteristic size of atoms.

Part IB

GENERAL ARRANGEMENTS

Structure of Part IB

Seventeen courses, including Computational Projects, are examined in Part IB. Some courses are lectured twice, to give the timetable greater flexibility: Quantum Mechanics and Fluid Dynamics are lectured in Michaelmas and Lent terms; Special Relativity is lectured in Easter term and Lent term. Complex Analysis and Complex Methods cover much the same material, from different points of view: students may attend either (or both) sets of lectures. Two courses (Optimisation and Numerical Analysis) can be taken in the Easter term of either the first year or the second year. Metric and Topological Spaces can also be taken in either Easter term, but it should be noted that some material will prove useful for Complex Analysis.

The Faculty Board guidance reagarding choice of courses in Part IB is as follows:

> *Part IB of the Mathematical Tripos provides a wide range of courses from which students should, in consultation with their Directors of Studies, make a selection based on their individual interests and preferred workload, bearing in mind that it is better to do a smaller number of courses thoroughly than to do many courses scrappily.*

Computational Projects

The lectures for Computational Projects will normally be attended in the Easter term of the first year, the Computational Projects themselves being done in the Michaelmas and Lent terms of the second year (or in the Long, Christmas and Easter vacations).

No questions on the Computational Projects are set on the written examination papers, credit for examination purposes being gained by the submission of notebooks. The maximum credit obtainable is 80 marks and 4 quality marks (alphas or betas), which is roughly the same as for a 16-lecture course. Credit obtained is added to the credit gained in the written examination.

Examination

Arrangements common to all examinations of the undergraduate Mathematical Tripos are given on pages 1 and 2 of this booklet.

Each of the four papers is divided into two sections. Candidates should attempt at most four questions from Section I and at most six questions in Section II.

The number of questions set on each course varies according to the number of lectures given, as shown:

Number of lectures	Section I	Section II
24	3	4
16	2	3
12	2	2
8	2	1

Questions on the different courses are distributed among the papers as specified in the following table. The letters S and L appearing the table denote a question in Section I and a question in Section II, respectively.

	Paper 1	Paper 2	Paper 3	Paper 4
Linear Algebra	L+S	L+S	L	L+S
Groups, Rings and Modules	L	L+S	L+S	L+S
Geometry	S	L	L+S	L
Analysis II	L	L+S	L+S	L+S
Metric and Topological Spaces	L	S	S	L
Complex Analysis	L+S*	L*	L	S
Complex Methods			S	L
Methods	L	L+S	L+S	L+S
Quantum Mechanics	L	L	L+S	S
Electromagnetism	L	L+S	L	S
Spectial Relativity	S	S		L
Fluid Dynamics	L+S	S	L	L
Numerical Analysis	S	L	L	S
Statistics	L+S	L	S	L
Optimization	S	S	L	L
Markov Chains	L	L	S	S

*On Paper 1 and Paper 2, Complex Analysis and Complex Methods are examined by means of common questions.

The following table shows the approximate mark, number of alphas, number of betas and merit mark of a typical candidate placed near the bottom of each class last year. This may be taken as a rough guide for the current year, but it is important to realise that borderlines may go up or down. At the lowest borderlines, individual consideration of candidates is always paramount, so no initial borderline is used and there are no typical candidates; *very few* candidates fail to achieve at least a third class.

2005	Borderline	Borderline candidate
	1/2(i)	329,12,9; 452
	2(i)/2(ii)	254,7,6; 321
	2(ii)/3	156,3,6; 195

LINEAR ALGEBRA — 24 lectures, Michaelmas term

Definition of a vector space (over $\mathbb{R}$ or $\mathbb{C}$), subspaces, the space spanned by a subset. Linear independence, bases, dimension. Direct sums and complementary subspaces. [3]

Linear maps, isomorphisms. Relation between rank and nullity. The space of linear maps from U to V, representation by matrices. Change of basis. Row rank and column rank. [4]

Determinant and trace of a square matrix. Determinant of a product of two matrices and of the inverse matrix. Determinant of an endomorphism. The adjugate matrix. [3]

Eigenvalues and eigenvectors. Diagonal and triangular forms. Characteristic and minimal polynomials. Cayley-Hamilton Theorem over $\mathbb{C}$. Algebraic and geometric multiplicity of eigenvalues. Statement and illustration of Jordan normal form. [4]

Dual of a finite-dimensional vector space, dual bases and maps. Matrix representation, rank and determinant of dual map [2]

Bilinear forms. Matrix representation, change of basis. Symmetric forms and their link with quadratic forms. Diagonalisation of quadratic forms. Law of inertia, classification by rank and signature. Complex Hermitian forms. [4]

Inner product spaces, orthonormal sets, orthogonal projection, $V = W \oplus W^{\perp}$. Gram-Schmidt orthogonalisation. Adjoints. Diagonalisation of Hermitian matrices. Orthogonality of eigenvectors and properties of eigenvalues. [4]

Appropriate books

C.W. Curtis *Linear Algebra: an introductory approach.* Springer 1984 (£38.50 hardback).
P.R. Halmos *Finite-dimensional vector spaces.* Springer 1974 (£31.50 hardback).
K. Hoffman and R. Kunze *Linear Algebra.* Prentice-Hall 1971 (£72.99 hardback).

GROUPS, RINGS AND MODULES — 24 lectures, Lent term

Groups

Basic concepts of group theory recalled from Algebra and Geometry. Normal subgroups, quotient groups and isomorphism theorems. Permutation groups. Groups acting on sets, permutation representations. Conjugacy classes, centralizers and normalizers. The centre of a group. Elementary properties of finite p-groups. Examples of finite linear groups and groups arising from geometry. Simplicity of A_5.

Sylow subgroups and Sylow theorems. Applications, groups of small order. [7]

Rings

Definition and examples of rings (commutative, with 1). Ideals, homomorphisms, quotient rings, isomorphism theorems. Prime and maximal ideals. Fields. The characteristic of a field. Field of fractions of an integral domain.

Factorization in rings; units, primes and irreducibles. Unique factorization in principal ideal domains, and in polynomial rings. Gauss' Lemma and Eisenstein's irreducibility criterion.

Rings $\mathbb{Z}[\alpha]$ of algebraic integers as subsets of $\mathbb{C}$ and quotients of $\mathbb{Z}[x]$. Examples of Euclidean domains and uniqueness and non-uniqueness of factorization. Factorization in the ring of Gaussian integers; representation of integers as sums of two squares.

Ideals in polynomial rings. Hilbert basis theorem. [9]

Modules

Definitions, examples of vector spaces, abelian groups and vector spaces with an endomorphism. Submodules, homomorphisms, quotient modules and direct sums. Equivalence of matrices, canonical form. Structure of finitely generated modules over Euclidean domains, applications to abelian groups and Jordan normal form. [8]

Appropriate books

P.M.Cohn *Classic Algebra.* Wiley, 2000 (£29.95 paperback).
P.J. Cameron *Introduction to Algebra.* OUP (£27 paperback).
J.B. Fraleigh *A First Course in abstract Algebra.* Addison Wesley, 2003 (£47.99 paperback).
B. Hartley and T.O. Hawkes *Rings, Modules and Linear Algebra: a further course in algebra.* Chapman and Hall, 1970 (out of print).
I. Herstein *Topics in Algebra.* John Wiley and Sons, 1975 (£45.99 hardback).
P.M. Neumann, G.A. Stoy and E.C. Thomson *Groups and Geometry.* OUP 1994 (£35.99 paperback).
M. Artin *Algebra.* Prentice Hall, 1991 (£53.99 hardback).

ANALYSIS II

24 lectures, Michaelmas term

Uniform convergence

The general principle of uniform convergence. A uniform limit of continuous functions is continuous. Uniform convergence and termwise integration and differentiation of series of real-valued functions. Local uniform convergence of power series. [3]

Uniform continuity and integration

Continuous functions on closed bounded intervals are uniformly continuous. Review of basic facts on Riemann integration (from Analysis I); proof that piecewise-continuous functions are integrable. Informal discussion of integration of complex-valued and $\mathbb{R}^n$-valued functions of one variable; proof that $\| \int_a^b f(x)\, dx\| \leq \int_a^b \|f(x)\|\, dx$. [2]

$\mathbb{R}^n$ as a normed space

Definition of a normed space. Examples, including the Euclidean norm on $\mathbb{R}^n$ and the uniform norm on $\mathcal{C}[a,b]$. Lipschitz mappings and Lipschitz equivalence of norms. The Bolzano–Weierstrass theorem in $\mathbb{R}^n$. Completeness. Open and closed sets. Continuity for functions between normed spaces. A continuous function on a closed bounded set in $\mathbb{R}^n$ is uniformly continuous and has closed bounded image. All norms on a finite-dimensional space are Lipschitz equivalent. [5]

Differentiation from $\mathbb{R}^m$ to $\mathbb{R}^n$

Definition of derivative as a linear map; elementary properties, the chain rule. Partial derivatives; continuous partial derivatives imply differentiability. Higher-order derivatives; symmetry of mixed partial derivatives (assumed continuous). Taylor's theorem. The mean value inequality. Path-connectedness for subsets of $\mathbb{R}^n$; a function having zero derivative on a path-connected open subset is constant. [6]

Metric spaces

Definition and examples. *Metrics used in Geometry*. Limits, continuity, balls, neighbourhoods, open and closed sets. [4]

The Contraction Mapping Theorem

The contraction mapping theorem. Applications including the inverse function theorem (proof of continuity of inverse function, statement of differentiability). Picard's solution of differential equations. [4]

Appropriate books

† J.C. Burkill and H. Burkill *A Second Course in Mathematical Analysis.* Cambridge University Press 2002 (£29.95 paperback).

A.F. Beardon *Limits: a new approach to real analysis.* Springer 1997 (£22.50 hardback).

† W. Rudin *Principles of Mathematical Analysis.* McGraw–Hill 1976 (£35.99 paperback).

W.A. Sutherland *Introduction to Metric and Topological Space.* Clarendon 1975 (£21.00 paperback).

A.J. White *Real Analysis: An Introduction.* Addison–Wesley 1968 (out of print).

T.W. Körner *A companion to analysis.* (AMS.2004).

METRIC AND TOPOLOGICAL SPACES

12 lectures, Easter term

Metrics

Definition and examples. Limits and continuity. Open sets and neighbourhoods. Characterizing limits and continuity using neighbourhoods and open sets. [3]

Topology

Definition of a topology. Metric topologies. Further examples. Neighbourhoods, closed sets, convergence and continuity. Hausdorff spaces. Homeomorphisms. Topological and non-topological properties. Completeness. Subspace, quotient and product topologies. [3]

Connectedness

Definition using open sets and integer-valued functions. Examples, including intervals. Components. The continuous image of a connected space is connected. Path-connectedness. Path-connected spaces are connected but not conversely. Connected open sets in Euclidean space are path-connected. [3]

Compactness

Definition using open covers. Examples: finite sets and $[0,1]$. Closed subsets of compact spaces are compact. Compact subsets of a Hausdorff space must be closed. The compact subsets of the real line. Continuous images of compact sets are compact. Quotient spaces. Continuous real-valued functions on a compact space are bounded and attain their bounds. The product of two compact spaces is compact. The compact subsets of Euclidean space. Sequential compactness. [3]

Appropriate books

† W.A. Sutherland *Introduction to metric and topological spaces.* Clarendon 1975 (£21.00 paperback).

A.J. White *Real analysis: an introduction.* Addison-Wesley 1968 (out of print).

B. Mendelson *Introduction to Topology.* Dover, 1990 (£5.27 paperback).

GEOMETRY — 16 lectures, Lent term

Parts of Analysis II will be found useful for this course.

Groups of rigid motions of Euclidean space. Rotation and reflection groups in two and three dimensions. Lengths of curves. [2]

Spherical geometry: spherical lines, spherical triangles and the Gauss-Bonnet theorem. Stereographic projection and Möbius transformations. [3]

Triangulations of the sphere and the torus, Euler number. [1]

Riemannian metrics on open subsets of the plane. The hyperbolic plane. Poincaré models and their metrics. The isometry group. Hyperbolic triangles and the Gauss-Bonnet theorem. The hyperboloid model. [4]

Embedded surfaces in $\mathbb{R}^3$. The first fundamental form. Length and area. Examples. [1]

Length and energy. Geodesics for general Riemannian metrics as stationary points of the energy. First variation of the energy and geodesics as solutions of the corresponding Euler-Lagrange equations. Geodesic polar coordinates (informal proof of existence). Surfaces of revolution. [2]

The second fundamental form and Gaussian curvature. For metrics of the form $du^2 + G(u,v)dv^2$, expression of the curvature as $\sqrt{G_{uu}}/\sqrt{G}$. Abstract smooth surfaces and isometries. Euler numbers and statement of Gauss-Bonnet theorem, examples and applications. [3]

Appropriate books

M. Do Carmo *Differential Geometry of Curves and Surfaces.* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1976 (£42.99 hardback).

V.V. Nikulin and I. Shafarevich *Geometries and Groups.* Springer, 1987 (£44.00 paperback).

A. Pressley *Elementary Differential Geometry.* Springer Undergraduate Mathematics Series, Springer-Verlag London Ltd., 2001 (£19.00 paperback).

E. Rees *Notes on Geometry.* Springer, 1983 (£18.50 paperback).

J.R. Weeks *The Shape of Space.* Dekker, 2002 (£37.50 paperback).

COMPLEX ANALYSIS — 16 lectures, Lent term

Analytic functions
Complex differentiation and the Cauchy-Riemann equations. Examples. Conformal mappings. Informal discussion of branch points, examples of log z and z^c. [3]

Contour integration and Cauchy's theorem
Contour integration (for piecewise continuously differentiable curves). Statement and proof of Cauchy's theorem for star domains. Cauchy's integral formula, maximum modulus theorem, Liouville's theorem, fundamental theorem of algebra. Morera's theorem. [4]

Expansions and singularities
Uniform convergence of analytic functions; local uniform convergence. Differentiability of a power series. Taylor and Laurent expansions. Principle of isolated zeros. Residue at an isolated singularity. Classification of isolated singularities. [4]

The residue theorem
Winding numbers. Residue theorem. Jordan's lemma. Evaluation of definite integrals by contour integration. Rouché's theorem, principle of the argument. Open mapping theorem. [3]

Fourier transforms
Definition of Fourier transform. Examples of computation. Discussion of Fourier tranforms of convolutions and derivatives, and the inversion formula. [2]

Appropriate books

L.V. Ahlfors *Complex Analysis.* McGraw–Hill 1978 (£119.99 hardback).
† A.F. Beardon *Complex Analysis.* Wiley (out of print).
† G.J.O. Jameson *A First Course on Complex Functions.* Chapman and Hall (out of print).
H.A. Priestley *Introduction to Complex Analysis.* Oxford University Press 2003 (£19.95 paperback).
I. Stewart and D. Tall *Complex Analysis.* Cambridge University Press 1983 (£27.00 paperback).

COMPLEX METHODS — 16 lectures, Lent term

Analytic functions
Definition of an analytic function. Cauchy-Riemann equations. Analytic functions as conformal mappings; examples. Application to the solutions of Laplace's equation in various domains. Discussion of $\log z$ and z^a. [5]

Contour integration and Cauchy's Theorem
[*Proofs of theorems in this section will not be examined in this course.*]
Domains, contours, contour integrals. Cauchy's theorem and Cauchy's integral formula. Taylor and Laurent series. Zeros, poles and essential singularities. [3]

Residue calculus
Residue theorem, calculus of residues. Jordan's lemma. Evaluation of definite integrals by contour integration. [4]

Fourier transforms
Definition and simple properties. The inversion formula. Applications to differential equations. Computation of transforms by contour integration. Transforms of step and delta functions.

Convolution. Parseval's formula. Application to time-invariant linear 'input' and 'output' systems. Causality. Frequency analysis of signals. [4]

Appropriate books

M.J. Ablowitz and A.S. Fokas *Complex variables: introduction and applications.* CUP 2003 (£65.00).
G. Arfken and H. Weber *Mathematical Methods for Physicists.* Harcourt Academic 2001 (£38.95 paperback).
G. J. O. Jameson *A First Course in Complex Functions.* Chapman and Hall 1970 (out of print).
T. Needham *Visual complex analysis.* Clarendon 1998 (£28.50 paperback).
† H.A. Priestley *Introduction to Complex Analysis.* Clarendon 1990 (out of print).
† I. Stewart and D. Tall *Complex Analysis (the hitchhiker's guide to the plane).* Cambridge University Press 1983 (£27.00 paperback).

METHODS

24 lectures, Michaelmas term

Fourier series and the wave equation

Periodic functions. Fourier series. Parseval's theorem. Wave equation for a string. Separation of variables. Normal modes for a string of finite length. Energy. Solution of form $f(x+ct)+g(x-ct)$. Wave reflection and transmission. [4]

Ordinary differential equations

Equations of the second order; initial value problems and problems with two fixed end points; solution using Green's function: notion of Green's function as an inverse operator. The Sturm–Liouville equation; eigenfunction and eigenvalues: reality of eigenvalues and orthogonality of eigenfunctions; eigenfunction expansions (Fourier series as prototype), approximation in mean square, statement of completeness; expansion of δ-function and Green's function. [6]

Laplace's equation

Separation of variables in Cartesians, plane polar and spherical polar co-ordinates, solutions in spherical polars for axisymmetric systems only. Legendre's equation and solutions as Legendre polynomials (statement only); orthogonality; forms of P_0, P_1 and P_2. [3]

Calculus of variations

Stationary points of $f(x_1,\ldots,x_n)$: necessary and sufficient conditions for the free case; Lagrange multipliers, necessary conditions for the constrained case. Euler–Lagrange equations, functional derivatives, first integrals; use of Lagrange multipliers (statement only). Examples e.g. Fermat's principle. [5]

Cartesian tensors in $\mathbb{R}^3$

Tensor transformation laws, addition, multiplication, contraction, the quotient theorem. Isotropic second and third rank tensors. Symmetric and antisymmetric tensors. Revision of principal axes and diagonalization. Examples e.g. inertia, stress, conductivity. [6]

Appropriate books

G. Arfken and H. Weber *Mathematical Methods for Physicists.* Harcourt Academic 2001 (£38.95 hardback).

M.L. Boas *Mathematical Methods in the Physical Sciences.* Wiley 1983 (£34.50 hardback).

H. Jeffreys *Cartesian Tensors.* Cambridge University Press 1931 (out of print).

H. Jeffreys and B.S. Jeffreys *Methods of Mathematical Physics.* Cambridge University Press 1999 (£25.95 paperback).

J. Mathews and R.L. Walker *Mathematical Methods of Physics.* Benjamin/Cummings 1970 (£68.99 hardback).

K. F. Riley, M. P. Hobson, and S.J. Bence *Mathematical Methods for Physics and Engineering: a comprehensive guide.* Cambridge University Press 2002 (£29.95 paperback).

B. Spain *Tensor Calculus.* Dover 2003 (£8.95 paperback).

QUANTUM MECHANICS

16 lectures, Michaelmas and Lent terms

Physical background

Photoelectric effect. Electrons in atoms and line spectra. Particle diffraction. [1]

Schrödinger equation and solutions

De Broglie waves. Schrödinger equation. Superposition principle. Probability interpretation, density and current. [2]

Stationary states. Free particle, Gaussian wave packet. Motion in 1-dimensional potentials, parity. Potential step, square well and barrier. Harmonic oscillator. [4]

Observables and expectation values

Position and momentum operators and expectation values. Canonical commutation relations. Uncertainty principle. [2]

Observables and Hermitian operators. Eigenvalues and eigenfunctions. Formula for expectation value. [2]

Hydrogen atom

Spherically symmetric wave functions for spherical well and hydrogen atom.

Orbital angular momentum operators. General solution of hydrogen atom. [5]

Appropriate books

Fenyman, Leighton and Sands *vol. 3 Ch 1-3 of the Fenyman lectures on Physics.* Addison-Wesley 1970 (£87.99 paperback).

† S. Gasiorowicz *Quantum Physics.* Wiley 2003 (£34.95 hardback).

P.V. Landshoff, A.J.F. Metherell and W.G Rees *Essential Quantum Physics.* Cambridge University Press 1997 (£21.95 paperback).

† A.I.M. Rae *Quantum Mechanics.* Institute of Physics Publishing 2002 (£16.99 paperback).

L.I. Schiff *Quantum Mechanics.* McGraw Hill 1968 (£38.99 hardback).

ELECTROMAGNETISM — 16 lectures, Lent term

Introduction to Maxwell's equations

Electric and magnetic fields, charge, current. Maxwell's equations and the Lorentz force. Charge conservation. Integral form of Maxwell's equations and their interpretation. Scalar and vector potentials; gauge transformations. [3]

Electrostatics

Point charges and the inverse square law, line and surface charges, dipoles. Electrostatic energy. Gauss's law applied to spherically symmetric and cylindrically symmetric charge distributions. Plane parallel capacitor. [3]

Steady currents

Ohm's law, flow of steady currents. Magnetic fields due to steady currents, simple examples treated by means of Ampère's equation. Vector potential due to a general current distribution, the Biot–Savart law. Magnetic dipoles. Lorentz force on steady current distributions and force between current-carrying wires. [4]

Electromagnetic induction

Faraday's law of induction for fixed and moving circuits; simple dynamo. [2]

Electromagnetic waves

Electromagnetic energy and Poynting vector. Plane electromagnetic waves in vacuum, polarisation. Reflection at a plane conducting surface. [4]

Appropriate books

W.N. Cottingham and D.A. Greenwood *Electricity and Magnetism.* Cambridge University Press 1991 (£17.95 paperback).
R. Feynman, R. Leighton and M. Sands *The Feynman Lectures on Physics, Vol 2.* Addison–Wesley 1970 (£87.99 paperback).
† P. Lorrain and D. Corson *Electromagnetism, Principles and Applications.* Freeman 1990 (£47.99 paperback).
J.R. Reitz, F.J. Milford and R.W. Christy *Foundations of Electromagnetic Theory.* Addison-Wesley 1993 (£46.99 hardback).
D.J. Griffiths *Introduction to Electrodynamics.* Prentice–Hall 1999 (£42.99 paperback).

SPECIAL RELATIVITY — 8 lectures, Easter and Lent terms

[Lecturers should use the signature convention $(+\,-\,-\,-)$.]

Space and time

The principle of relativity. Relativity and simultaneity. The invariant interval. Lorentz transformations in $(1+1)$-dimensional spacetime. Time dilation and muon decay. Length contraction. The Minkowski metric for $(1+1)$-dimensional spacetime. [4]

4–vectors

Lorentz transformations in $(3+1)$ dimensions. 4–vectors and Lorentz invariants. Proper time. 4–velocity and 4–momentum. Conservation of 4–momentum in radioactive decay. [4]

Appropriate books

G.F.R. Ellis and R.M. Williams *Flat and Curved Space-times.* Oxford University Press 2000 (£24.95 paperback).
† W. Rindler *Introduction to Special Relativity.* Oxford University Press 1991 (£19.99 paperback).
W. Rindler *Relativity: special, general and cosmological.* OUP 2001 (£24.95 paperback).
† E.F. Taylor and J.A. Wheeler *Spacetime Physics: introduction to special relativity.* Freeman 1992 (£29.99 paperback).

FLUID DYNAMICS — 16 lectures, Michaelmas and Lent terms

Kinematics
Continuum fields: density and velocity. Flow visualisation: particle paths, streamlines and dye streak-lines. Material time derivative. Conservation of mass and the kinematic boundary condition at a moving boundary. Incompressibility and streamfunctions. [3]

Dynamics
Surface and volume forces; pressure in frictionless fluids. The Euler momentum equation. Applications of the momentum integral.

Bernoulli's theorem for steady flows with potential forces; applications.

Vorticity, vorticity equation, vortex line stretching, irrotational flow remains irrotational, ∗Kelvin's circulation theorem∗. [4]

Potential flows
Velocity potential; Laplace's equation. Examples of solutions in spherical and cylindrical geometry by separation of variables.

Expression for pressure in time-dependent potential flows with potential forces; applications. Spherical bubbles; small oscillations, collapse of a void. Translating sphere and inertial reaction to acceleration, ∗effects of friction∗. Fluid kinetic energy for translating sphere.

Lift on an arbitrary 2D aerofoil with circulation, ∗generation of circulation.∗ [6]

Interfacial flows
Governing equations and boundary conditions. Linear water waves: dispersion relation, deep and shallow water, particle paths, standing waves in a container. Rayleigh-Taylor instability.

River flows: over a bump, out of a lake over a broad weir, ∗hydraulic jumps∗. [3]

Appropriate books

† D.J. Acheson *Elementary Fluid Dynamics.* Oxford University Press 1990 (£23.50 paperback).
G.K. Batchelor *An Introduction to Fluid Dynamics.* Cambridge University Press 2000 (£19.95 paperback).
E. Guyon, J.P. Hulin, L. Petit and C.D. Mitescu *Physical Hydrodynamics.* Oxford University Press 2000 (£59.50 hardback, £29.50 paperback).
G.M. Homsey et al. *Multi-Media Fluid Mechanics.* Cambridge University Press 2000 (CD-ROM for Windows or Macintosh, £14.95).
M.J. Lighthill *An Informal Introduction to Theoretical Fluid Mechanics.* Oxford University Press (out of print).
† A.R. Paterson *A First Course in Fluid Dynamics.* Cambridge University Press 1983 (£27.95 paperback).
M. van Dyke *An Album of Fluid Motion.* Parabolic Press (out of print).

NUMERICAL ANALYSIS — 12 lectures, Easter term

Linear equations and least squares calculations
LU triangular factorization of matrices. Relation to Gaussian elimination. Column pivoting. Factorizations of symmetric and band matrices. Iterative methods for linear equations. QR factorization of rectangular matrices by Gram–Schmidt, Givens and Householder techniques. Application to linear least squares calculations. [6]

Polynomial approximation
Interpolation by polynomials. Divided differences of functions and relations to derivatives. Orthogonal polynomials and their recurrence relations. Gaussian quadrature formulae. Peano kernel theorem and applications. [6]

Appropriate books

† S.D. Conte and C. de Boor *Elementary Numerical Analysis: an algorithmic approach.* McGraw–Hill 1980 (out of print).
G.H. Golub and C. Van Loan *Matrix Computations.* Johns Hopkins University Press 1996 (out of print).
M.J.D. Powell *Approximation Theory and Methods.* Cambridge University Press 1981 (£25.95 paperback).

STATISTICS — 16 lectures, Lent term

Estimation
Review of distribution and density functions, parametric families, sufficiency, Rao–Blackwell theorem, factorization criterion, and examples: binomial, Poisson, gamma. Maximum likelihood estimation. Confidence intervals. Use of prior distributions and Bayesian inference. [5]

Hypothesis testing
Simple examples of hypothesis testing, null and alternative hypothesis, critical region, size, power, type I and type II errors, Neyman–Pearson lemma. Significance level of outcome. Uniformly most powerful tests. Likelihood ratio, and use of likelihood ratio to construct test statistics for composite hypotheses. Generalized likelihood-ratio test. Goodness-of-fit tests and contingency tables. [6]

Linear normal models
The χ^2, t and F distributions, joint distribution of sample mean and variance, Student's t-test, F-test for equality of two variances. One-way analysis of variance. [3]

Linear regression and least squares
Simple examples. ∗Use of software∗. [2]

Appropriate books

D.A.Berry and B.W. Lindgren *Statistics, Theory and Methods.* Wadsworth 1996 (£36.00 paperback).
G. Casella and J.O. Berger *Statistical Inference.* Duxbury 2002 (£81.00 hardback).
M.H. DeGroot *Probability and Statistics.* Addison-Wesley 2002 (£42.99 paperback).
† J.A. Rice *Mathematical Statistics and Data Analysis.* Duxbury 1995 (out of print).

MARKOV CHAINS — 12 lectures, Michaelmas term

Discrete-time chains
Definition and basic properties, the transition matrix. Calculation of n-step transition probabilities. Communicating classes, closed classes, absorption, irreducibility. Calculation of hitting probabilities and mean hitting times; survival probability for birth and death chains. Stopping times and statement of the strong Markov property. [5]

Recurrence and transience; equivalence of transience and summability of n-step transition probabilities; equivalence of recurrence and certainty of return. Recurrence as a class property, relation with closed classes. Simple random walks in dimensions one, two and three. [3]

Invariant distributions, statement of existence and uniqueness up to constant multiples. Mean return time, positive recurrence; equivalence of positive recurrence and the existence of an invariant distribution. Convergence to equilibrium for irreducible, positive recurrent, aperiodic chains *and proof by coupling*. Long-run proportion of time spent in given state. [3]

Time reversal, detailed balance, reversibility; random walk on a graph. [1]

Appropriate books

G.R. Grimmett and D.R. Stirzaker *Probability and Random Processes.* OUP 2001 (£29.95 paperback).
J.R. Norris *Markov Chains.* Cambridge University Press 1997 (£20.95 paperback).

OPTIMISATION — 12 lectures, Easter term

Lagrangian methods
General formulation of constrained problems; the Lagrangian sufficiency theorem. Interpretation of Lagrange multipliers as shadow prices. Examples. [2]

Linear programming in the nondegenerate case
Convexity of feasible region; sufficiency of extreme points. Standardization of problems, slack variables, equivalence of extreme points and basic solutions. The primal simplex algorithm, artificial variables, the two-phase method. Practical use of the algorithm; the tableau. Examples. The dual linear problem, duality theorem in a standardized case, complementary slackness, dual variables and their interpretation as shadow prices. Relationship of the primal simplex algorithm to dual problem. Two person zero-sum games. [6]

Network problems
The Ford-Fulkerson algorithm and the max-flow min-cut theorems in the rational case. Network flows with costs, the transportation algorithm, relationship of dual variables with nodes. Examples. Conditions for optimality in more general networks; ∗the simplex-on-a-graph algorithm∗. [3]

Practice and applications
∗Efficiency of algorithms∗. The formulation of simple practical and combinatorial problems as linear programming or network problems. [1]

Appropriate books

† M.S. Bazaraa, J.J. Jarvis and H.D. Sherali *Linear Programming and Network Flows.* Wiley 1990 (£80.95 hardback).

† D. Luenberger *Linear and Nonlinear Programming.* Addison–Wesley 1984 (out of print).

R.J. Vanderbei *Linear programming: foundations and extensions.* Kluwer 2001 (£61.50 hardback).

COMPUTATIONAL PROJECTS — 6 lectures, Easter term plus project work

The lectures are given in the Easter Full Term of the Part IA year and are accompanied by introductory practical classes. Students register for these classes: arrangements will be announced during the lectures.

The Faculty publishes a projects booklet by the end of July preceding the Part IB year (i.e., shortly after the end of Term 3). This contains details of the projects and information about course administration. The booklet is available on the Faculty website at `http://www.maths.cam.ac.uk/catam/`

Full credit nmay obtained from the submission of the two core projects and a further two projects. Once the booklet is available, these projects may be undertaken at any time up to the submission deadlines, which are near the start of the Full Lent Term in the IB year for the two core projects and near the start of the Full Easter Term in the IB year for the two additional projects.

The lectures will cover some or all of the topics listed below.

Introduction to course
Examples of the use of computers in mathematics. Overview of course, including administration, marks and computing facilities. Introduction to the projects.

Using Windows and CCATSL elements

Introduction to programming in C (editor, compiler and graphics)
Structure of a program, and programming techniques; manipulation of arrays, iteration and recursion. Basic C commands. Complexity of an algorithm.

Introduction to software libraries and packages
How to call mathematical and graphical software libraries from C with illustrations, for example, numerical solution of ordinary differential equations.

Example project
The key points of how to complete a project should be covered by illustration.

Appropriate books

† Faculty of Mathematics, University of Cambridge *Learning to use C and the CATAM software library.* Available online or from CMS Reception (£2.00).

† Faculty of Mathematics, University of Cambridge *The CATAM Software Library.* Available from online or from CMS Reception (£4.00).

† S.D. Conte and C. de Boor *Elementary Numerical Analysis.* McGraw–Hill 1980 (out of print).

C.F. Gerald and P.O. Wheatley *Applied Numerical Analysis.* 6th edition, Addison–Wesley 1999 (£67.99 hardback). 7th ed. entitled 'Numerical Analysis' due for publication September 2003

R.D. Harding and D.A. Quinney *A Simple Introduction to Numerical Analysis, Volume 1.* Institute of Physics Publishing 1986 (£19.95 paperback).

W.H. Press, B.P. Flannery, S.A. Teukolsky and W.T. Vetterling *Numerical Recipes: the Art of Scientific Computing.* Various editions based on different computer languages, Cambridge University Press 2002 (£45.00 hardback).

Part II

GENERAL ARRANGEMENTS

Structure of Part II

There are two types of lecture courses in Part II: C courses and D courses. C courses are intended to be straightforward and accessible, and of general interest, whereas D courses are intended to be more demanding. The Faculty Board recommend that students who have not obtained at least a good second class in Part IB should include a significant number of type C courses amongst those they choose.

There are 10 C courses and 25 D courses. All C courses are 24 lectures; of the D courses, 18 are 24 lectures and 7 are 16 lectures. The complete list of courses is as follows (the asterisk denotes a 16-lecture course):

C courses	**D courses**	
Number Theory	Representation Theory	Stochastic Financial Models
Topics in Analysis	Galois Theory	Partial Differential Equations
Geometry and Groups	Algebraic Topology	Principles of Quantum Mechanics
Coding and Cryptography	Linear Analysis	Applications of Quantum Mechanics
Statistical Modelling	Riemann Surfaces	*Statistical Physics
Mathematical Biology	Differential Geometry	*Electrodynamics
Dynamical Systems	Set Theory and Logic	*General Relativity
Further Complex Methods	Graph Theory	*Asymptotic Methods
Classical Dynamics	*Number Fields	Fluid Dynamics
Cosmology	Probability and Measure	Waves
	Applied Probability	Integrable Systems
	*Optimization and Control	Numerical Analysis
	Principles of Statistics	

Computational Projects

In addition to the lectured courses, there is a Computational Projects course. No questions on the Computational Projects are set on the written examination papers, credit for examination purposes being gained by the submission of notebooks. The maximum credit obtainable is 60 marks and 3 quality marks (alphas or betas), which is the same as that available for a 16-lecture course. Credit obtained on the Computational Projects is added to that gained in the written examination.

Examinations

Arrangements common to all examinations of the undergraduate Mathematical Tripos are given on pages 1 and 2 of this booklet.

There are no restrictions on the number or type of courses that may be presented for examination. The Faculty Board has recommended to the examiners that no distinction be made, for classification purposes, between quality marks obtained on the Section II questions for C course questions and those obtained for D course questions.

Candidates should answer no more than six questions in Section I on each paper; there is no restriction on the number of questions in Section II that may be answered.

The number of questions set on each course is determined by the type and length of the course, as shown in the following table.

	Section I	Section II
C course (24 lectures)	4	2
D course, 24 lectures	—	4
D course, 16 lectures	—	3

In Section I of each paper, there are 10 questions, one on each C course.

In Section II of each paper, there are 5 questions on C courses, one question on each of the 18 24-lecture D courses and either one question or no questions on each of the 7 16-lecture D courses, giving a total of 28 or 29 questions.

The distribution in Section II of the C course questions and the 16-lecture D course questions is shown in the following table.

C courses	P1	P2	P3	P4
Number Theory			*	*
Topics in Analysis	*	*		
Geometry and Groups	*			*
Coding and Cryptography		*	*	
Statistical Modelling	*			*
Mathematical Biology		*	*	
Dynamical Systems	*	*		
Further Complex Methods			*	*
Classical Dynamics	*		*	
Cosmology		*		*

16-lecture D courses	P1	P2	P3	P4
Number Fields	*	*		*
Optimization and Control		*	*	*
Asymptotic Methods	*		*	*
Integrable Systems	*	*	*	
Statistical Physics		*	*	*
Electrodynamics	*		*	*
General Relativity	*	*		*

The following table shows the approximate mark, number of alphas, number of betas and merit mark of a typical candidate placed near the bottom of each class last year. This may be taken as a rough guide for the current year, but it is important to realise that borderlines may go up or down. At the lowest borderlines, individual consideration of candidates is always paramount, so there are no typical candidates; *very few* candidates fail to achieve at least a third class.

2005	Borderline	Borderline candidate
	1/2(i)	334,12,11; 463
	2(i)/2(ii)	225,7,7; 295
	2(ii)/3	161,4,5; 204

NUMBER THEORY (C) — 24 lectures, Lent term

No specific prerequisites.

Review from Part IA Numbers and Sets: Euclid's Algorithm, prime numbers, fundamental theorem of arithmetic. Congruences. The theorems of Fermat and Euler. [2]

Chinese remainder theorem. Lagrange's theorem. Primitive roots to an odd prime power modulus. [3]

The mod-p field, quadratic residues and non-residues, Legendre's symbol. Euler's criterion. Gauss' lemma, quadratic reciprocity. [2]

Proof of the law of quadratic reciprocity. The Jacobi symbol. [1]

Binary quadratic forms. Discriminants. Standard form. Representation of primes. [5]

Distribution of the primes. Divergence of $\sum_p p^{-1}$. The Riemann zeta-function and Dirichlet series. Statement of the prime number theorem and of Dirichlet's theorem on primes in an arithmetic progression. Legendre's formula. Bertrand's postulate. [4]

Continued fractions. Pell's equation. [3]

Primality testing. Fermat, Euler and strong pseudo-primes. [2]

Factorization. Fermat factorization, factor bases, the continued-fraction method. Pollard's method. [2]

Appropriate books

A. Baker *A Concise Introduction to the Theory of Numbers.* Cambridge University Press 1984 (£14.95 paperback).

G.H. Hardy and E.M. Wright *An Introduction to the Theory of Numbers.* Oxford University Press 1979 (£18.50 paperback).

N. Koblitz *A Course in Number Theory and Cryptography.* Springer 1994 (£22.50 hardback).

T. Nagell *Introduction to Number Theory.* OUP 1981 (£16.00 hardback).

I. Niven, H.S. Zuckerman and H.L. Montgomery *An Introduction to the Theory of Numbers.* Wiley 1991 (£71.95 hardback).

H. Riesel *Prime Numbers and Computer Methods for Factorization.* Birkhauser 1994 (£69.50 hardback).

TOPICS IN ANALYSIS (C) — 24 lectures, Michaelmas term

Analysis courses from IB will be helpful, but it is intended to introduce and develop concepts of analysis as required.

Discussion of metric spaces; compactness and completeness. Brouwer's fixed point theorem. Proof(s) in two dimensions. Equivalent formulations, and applications. The degree of a map. The fundamental theorem of algebra, the Argument Principle for continuous functions, and a topological version of Rouché's Theorem. [6]

The Weierstrass Approximation Theorem. Chebychev polynomials and best uniform approximation. Gaussian quadrature converges for all continuous functions. Review of basic properties of analytic functions. Runge's Theorem on the polynomial approximation of analytic functions. [8]

Liouville's proof of the existence of transcendentals. The irrationality of e and π. The continued fraction expansion of real numbers; the continued fraction expansion of e. [4]

Review of countability, topological spaces, and the properties of compact Hausdorff spaces. The Baire category theorem for a complete metric space. Applications. [6]

Appropriate books

A.F. Beardon *Complex Analysis: the Argument Principle in Analysis and Topology.* John Wiley & Sons, 1979 (Out of print).

E.W. Cheney *Introduction to Approximation Theory.* AMS, 1999 (£21.00 hardback).

G.H. Hardy and E.M. Wright *An Introduction to the Theory of Numbers.* Clarendon Press, Oxford, fifth edition, reprinted 1989 (£28.50 paperback).

T. Sheil-Small *Complex Polynomials.* Cambridge University. Press, 2002 (£70.00 hardback).

GEOMETRY AND GROUPS (C) — 24 lectures, Lent term

Analysis II and Geometry are useful, but any concepts required from these courses will be introduced and developed in the course.

A brief discussion of the Platonic solids. The identification of the symmetry groups of the Platonic solids with the finite subgroups of SO(3). [4]

The upper half-plane and disc models of the hyperbolic plane. The hyperbolic metric; a comparison of the growth of circumference and area in Euclidean and hyperbolic geometry. The identification of isometries of the upper half-plane with real Möbius maps, and the classification of isometries of the hyperbolic plane. The Modular group $SL(2, \mathbb{Z})$ and its action on the upper half-plane. The tesselation of hyperbolic plane by ideal triangles. [6]

Group actions: discrete groups, properly discontinuous group actions. Quotients by group actions and fundamental domains. Brief discussion of Escher's work as illustrations of tesselations of the hyperbolic plane, and of Euclidean crystallographic groups. [5]

Examples of fractals, including Cantor's middle-third set and similar constructions. Cantor sets, and non-rectifiable curves, as limit sets of discrete Möbius groups. Fractals obtained dynamically. The Hausdorff dimension of a set, computations of examples. [5]

The upper half-space of $\mathbb{R}^3$ as hyperbolic three-space; the identification of the group of Möbius maps as the isometries of hyperbolic three-space, and of the finite groups of Möbius maps with the the finite subgroups of SO(3). *Discussion of Euclidean crystallographic groups in higher dimensions.* [4]

Appropriate books

A.F. Beardon *The geometry of discrete groups.* Springer-Verlag, Graduate Texts No. 91, 1983 (£?).
K. Falconer *Fractal Geometry.* John Wiley & Sons, 1990 (£85.00 Hardback).
R.C. Lyndon *Groups and Geometry, LMS Lecture Notes 101.* Cambridge University Press, 1985 (£?).
V.V. Nikulin and I.R. Shafarevich *Geometries and Groups.* Springer-Verlag, 1987 (£?).

CODING AND CRYPTOGRAPHY (C) — 24 lectures, Michaelmas term

Linear Analysis is useful and Groups, Rings and Modules is very useful.

Introduction to communication channels, coding and channel capacity. [1]

Data compression; decipherability. Kraft's inequality. Huffman and Shannon-Fano coding. [2]

Applications to gambling and the stock market. [1]

Codes, error detection and correction, Hamming distance. Examples: simple parity, repetition, Hamming's original [7,16] code. Maximum likelihood decoding. The Hamming and V-G-S bounds. Entropy. [4]

Shannon's coding theorems. Information rate of a Bernoulli source. Capacity of a memoryless binary symmetric channel. [3]

Linear codes, weight, generator matrices, parity checks. Syndrome decoding. Dual codes. Examples: Hamming, Reed-Muller. [2]

Cyclic codes. Discussion without proofs of the abstract algebra (finite fields, polynomial rings and ideals) required. Generator and check polynomials. BCH codes. Error-locator polynomial and decoding. Recurrence (shift-register) sequences. Berlekamp-Massey algorithm. [4]

Introduction to cryptography. Unicity distance, one-time pad. Shift-register based pseudo-random sequences. Linear complexity. Simple cipher machines. [1]

Public-key cryptography. Secrecy and authentication. The RSA system. The discrete logarithm problem and el Gamal (DSA) signatures. Bit commitment and coin tossing. [4]

Quantum cryptography; introduction to the BB84 protocol. [2]

Appropriate books

† G.M. Goldie and R.G.E. Pinch *Communication Theory* . Cambridge University Press 1991 (£25.95 paperback).
D. Welsh *Codes and Cryptography.* OUP 1988 (£25.95 paperback).
T.M. Cover and J.A. Thomas *Elements of Information Theory.* Wiley 1991 (£58.50 hardback).

STATISTICAL MODELLING (C) — 24 lectures, Lent term

Part IB Statistics is essential. About two thirds of this course will be lectures, with the remaining hours as practical classes, using R in the CATAM system. R may be downloaded at no cost via http://www.stats.bris.ac.uk/R/.

Introduction to the statistical package R
The use of *R* for graphical summaries of data, eg histograms, and for classical tests such as t-tests and F-tests. [2]

Review of likelihood function
Asymptotic distribution of the maximum likelihood estimator (outline only). Approximate confidence intervals for parameters. Asymptotic distribution of deviance, and of the reduction in deviance when fitting nested models. [5]

Linear models with normal errors
Review of the multivariate normal distribution. Least squares estimates and projection matrices. Construction of analysis of variance; use of F-tests. Orthogonality of sets of parameters. The definition of a factor, interactions between factors and the interpretation of interactions. [5]

Model criticism
Examination of residuals and leverages, qq-plots. [2]

Exponential families and generalized linear models (glm).
Iterative solution of the likelihood equation for the glm family. Regression for binomial data; use of logit and other link functions. Regression for count data: the Poisson distribution, its glm relationship to the multinomial distribution, and the application to contingency tables, both 2-way and 3-way. Conditional independence. Regression for the gamma distribution. [8]

Advanced modelling techniques
These may include any of the following *simulation, Bayesian analysis, non-linear regression, graphical modelling.* [2]

Appropriate books

A.J.Dobson *An Introduction to Generalized Linear Models.* Chapman and Hall 2002 (£17.49 Paperback).
J.Faraway *Practical Regression and Anova in R, 2002.* http://www.stat.lsa.umich.edu/~faraway/book/ (£?).
Y.Pawitan *In All Likelihood: Statistical Modelling and Inference Using Likelihood.* Oxford Science Publications 2001 (£40.50 Hardback).

MATHEMATICAL BIOLOGY (C) — 24 lectures, Michaelmas term

Population dynamics and epidemiology
Continuous and discrete population dynamics without spatial structure. Discrete stochastic models and continuum limits. Infectious disease models: transmission and control. Time delay models and models with age distribution. Logistic model, bifurcation to chaos. Models for interacting populations. [6]

Simple oscillatory reactions
Feedback control mechanism, oscillators and switches. Belousov-Zhabotinskii reactions. Perturbed and coupled Oscillators. [2]

Diffusion and reaction-diffusion systems
Simple random walk and derivation of the diffusion equation. Fundamental solutions for steady and unsteady diffusion. Models of animal dispersal, chemotaxis, cell potential and energy approach to diffusion. Fisher-Kolmogorov equation. Reaction-diffusion models with density-dependent diffusion. General conditions for diffusion-driven instability: linear stability analysis and evolution of spatial pattern. Numerical methods for solving reaction-diffusion equations. [10]

Modelling transcriptional control in gene networks
Statistical analysis of gene expression data. Stochastic models: a probabilistic model of a prokaryotic gene and its regulation. Discrete, continuous and delay neural networks. Mathematical approaches in one dimension: the Fokker-Planck equation, boundary conditions, chemical reaction model, eigenfunction methods. [6]

Appropriate books

J.D. Murray *Mathematical Biology (3rd edition), especially volume 1.* Springer, 2002 (£42.35 hardback).
J.M. Bower and H. Bolouri *(editors) Computational Modelling of Genetic and Biochemical Networks.* MIT Press, 2004 ($35.00).
C.W. Gardiner *Handbook of Stochastic Methods (2nd edition).* Springer, 2004 (£61.50 paperback).

DYNAMICAL SYSTEMS (C) — 24 lectures, Lent term

Analysis II is useful.

General introduction

The notion of a dynamical system and examples. Relationship between continuous systems and discrete systems. Reduction to autonomous systems. Existence and uniqueness for initial value problems (statement only), finite-time blowup, examples. Orbits, invariant sets, limit sets. Topological conjugacy of maps and equivalence of flows. [3]

Fixed points of flows

Linearization. Classification of fixed points in $\mathbb{R}^2$, Hamiltonian case. Effects of nonlinearity; hyperbolic and non-hyperbolic cases, stable and unstable manifolds in $\mathbb{R}^2$, stable-manifold and Hartmann–Grobman theorems (statements only). [3]

Stability

Lyapunov, quasi-asymptotic and asymptotic stability of invariant sets. Lyapunov and bounding functions. Lyapunov's 1st and 2nd theorems, La Salle's invariance principle. Local and global stability. [2]

Periodic orbits in $\mathbb{R}^2$

The Poincaré index; Dulac's criterion; the Poincaré–Bendixson theorem (*and proof*). Nearly Hamiltonian flows. Stability of periodic orbits, Floquet multiplier and Poincaré section. Examples: damped pendulum with torque, van der Pol oscillator. [5]

Bifurcations of flows and maps

Non-hyperbolicity and structural stability. Local bifurcations of fixed points: saddle-node, transcritical, pitchfork and Andronov-Hopf bifurcations. Construction of centre manifold and normal forms. Effects of symmetry and symmetry breaking. *Bifurcations of periodic orbits.* Fixed points and periodic points for maps. Bifurcations in 1-dimensional maps: saddle-node, period-doubling, transcritical and pitchfork bifurcations. Examples. *Hopf bifurcation for maps*. [5]

Chaos

Sensitive dependence on initial conditions, transitivity. Non-invertible maps of the interval, the sawtooth map, horseshoes, symbolic dynamics. Period three implies chaos, the occurrence of N-cycles, Sharkovsky's theorem (statement only). The tent map. The logistic map. Unimodal maps, renormalization, Feigenbaum's constant. [6]

Appropriate books

D.K. Arrowsmith and C.M. Place *Introduction to Dynamical Systems.* CUP 1990 (£31.95 paperback).
P.G. Drazin *Nonlinear Systems.* CUP1992 (£24.95 paperback).
† P.A. Glendinning *Stability, Instability and Chaos.* CUP1994 (£30.95 paperback, £70.00 hardback).
R. Grimshaw *An Introduction to Nonlinear Ordinary Differential Equations.* Blackwell Scientific 1990 (£49.00 hardback).
D.W. Jordan and P. Smith *Nonlinear Ordinary Differential Equations.* OUP 1999 (£24.50 paperback).
S. Wiggins *Introduction to Applied Nonlinear Dynamical Systems and Chaos.* Springer 1990 (£34.00 hardback).
J. Guckenheimer and P. Holmes *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields.* Springer, second edition 1986 (£37.50 hardback).

FURTHER COMPLEX METHODS (C) — 24 lectures, Michaelmas term

Complex Methods (or Complex Analysis) is essential.

Complex variable

Cauchy principal value of finite and infinite range improper integrals. Analyticity of a function defined by an integral (statement and discussion only). Analytic continuation.

Multivalued functions: definitions, branch points and cuts, integration. Inverse trigonometric functions as integrals. [6]

Ordinary differential equations in the complex plane

Classification of singularities of an ordinary differential equation, Exponents at a regular singular point. Nature of the solution near an isolated singularity by analytic continuation of solutions around the singularity. Classification of Fuchsian differential equations. Solution of differential equations by integral representation using Laplace and Fourier kernels; Airy equation as an example. [4]

Special functions

Gamma function, including Hankel representation; beta function; Riemann zeta function, *discussion of zeros and relation to $\pi(x)$ and the distribution of prime numbers*. The hypergeometric equation and hypergeometric functions. [6]

Transform methods

Revision of Fourier transforms. The Laplace transform: elementary properties; inversion formula; application to linear differential equations with constant coefficients. Other transform pairs (e.g. Hilbert transform). [5]

Elliptic integrals and Jacobi elliptic functions

Brief discussion of elliptic integrals. Definition and elementary properties (including periodicity) of sn, cn and dn. [3]

Appropriate books

† M.J. Ablowitz and A.S. Fokas *Complex Variables: Introduction and Applications.* CUP 2003 (£32 paperback).
† E.T. Whittaker and G.N. Watson *A course of modern analysis.* CUP 1996 (£27.95 paperback).
E.T. Copson *Functions of a Complex Variable.* Oxford University Press 1935 (out of print).
B. Spain and M.G. Smith *Functions of Mathematical Physics.* Van Nostrand 1970 (out of print).

CLASSICAL DYNAMICS (C) — 24 lectures, Michaelmas term

Part IB Methods is essential.

Review of Newtonian mechanics

Newton's second law. Motion of N particles under mutual interactions. Euclidean and Galilean symmetry. Conservation laws of momentum, angular momentum and energy. [2]

Lagrange's equations

Configuration space, generalized coordinates and velocities. Holonomic constraints. Lagrangian, Hamilton's principle and Euler-Lagrange equations of motion. Examples: N particles with potential forces, planar and spherical pendulum, charged particle in a background electromagnetic field, purely kinetic Lagrangians and geodesics. Ignorable coordinates. Symmetries and Noether's theorem. [5]

Quadratic Lagrangians, oscillations, normal modes. [1]

Motion of a rigid body

Kinematics of a rigid body. Angular momentum, kinetic energy, diagonalization of inertia tensor. Euler top, conserved angular momentum, Euler equations and their solution in terms of elliptic integrals. Lagrange top, steady precession, nutation. [6]

Hamilton's equations

Phase space. Hamiltonian and Hamilton's equations. Simple examples. Poisson brackets, conserved quantities. Principle of least action. Liouville theorem. Action and angle variables for closed orbits in 2-D phase space. Adiabatic invariants (proof not required). Mention of completely integrable systems, and their action-angle variables. [7]

Hamiltonian systems in nonlinear phase spaces, e.g. classical spin in a magnetic field. 2-D motion of ideal point vortices. *Connections between Lagrangian/Hamiltonian dynamics and quantum mechanics.* [3]

Appropriate books

† L.D. Landau and E.M. Lifshitz *Mechanics.* Butterworth-Heinemann 2002 (£34.99).
F. Scheck *Mechanics: from Newton's laws to deterministic chaos.* Springer 1999 (£38.50).
L.N. Hand and J.D. Finch *Analytical Mechanics.* CUP 1999 (£38.00 paperback).
H. Goldstein, C. Poole and J. Safko *Classical Mechanics.* Pearson 2002 (£34.99 paperback).
V.I. Arnold *Mathematical methods of classical mechanics.* Springer 1978 (£45.50 hardback).

COSMOLOGY (C) — 24 lectures, Lent term

Part IB Quantum Mechanics, Methods and Special Relativity are essential.

Introduction

The matter content of the universe. Hierarchy of atoms, nucleons and quarks. The four forces of nature, brief mention of the standard model and unification. Dark matter. [1]

The expanding universe

The Cosmological Principle: homogeneity and isotropy. Derivation of Hubble's law. Scale factor of the universe. Kinematic effects, including redshift. Horizon problem. [2]

Simple Newtonian account of an expanding fluid. Derivation of Friedmann and Raychaudhuri equations. Possible worlds: open, closed, and flat models. Flatness problem. Age of the universe. The late universe. Inflation and the problems of the standard cosmology. [4]

Statistical physics

Particles in a box, statistical physics and definition of entropy. Laws of thermodynamics. Temperature and chemical potential. Ideal gases of bosons and fermions: classical and relativistic limits. Fermi degeneracy pressure. [3]

Stars and gravitational collapse

Thermostatic equilibrium and the virial theorem. Qualitative evolution of stars. White dwarfs and neutron stars as made up of self-gravitating degenerate Fermi gases. Mention of black holes. [4]

Thermal history of the universe

Cosmic microwave background. Recombination and photon decoupling. Planck spectrum. Conservation of entropy and particle number. Helium and primordial nucleosynthesis. Mention of matter/antimatter asymmetry and the early universe. [5]

Origin of structure in the universe

Definition of density perturbation. Cosmological perturbation equation derived. Simple growing mode solution. Fourier decomposition, mode evolution and power spectrum. Non-zero pressure and the Jeans length. [4]

Discussion of anisotropies in the cosmic microwave background. Simple description of inflationary fluctuations. [1]

Appropriate books

E.R. Harrison *Cosmology: The Science of the Universe.* CUP 2000 (£35.00 hardback).
A. Liddle *An Introduction to Modern Cosmology.* Wiley 1998 (£17.99 paperback).
E.W. Kolb and M.S. Turner *The Early Universe.* Perseus 1994 (£40.99 paperback).
F. Mandl *Statistical Physics.* Wiley 1988 (£27.50 paperback).
A.C. Phillips *The Physics of Stars.* Wiley 1999 (£27.50 paperback).
S. Weinberg *The First Three Minutes.* Basic Books 1993 (£8.50 paperback).

SET THEORY AND LOGIC (D) — 24 lectures, Lent term

No specific prerequisites.

Ordinals and cardinals
Well-orderings and order-types. Examples of countable ordinals. Uncountable ordinals and Hartogs' lemma. Induction and recursion for ordinals. Ordinal arithmetic. Cardinals; the hierarchy of alephs. Cardinal arithmetic. [5]

Posets and zorn's Lemma
Partially ordered sets; Hasse diagrams, chains, maximal elements. Lattices and Boolean algebras. Complete and chain-complete posets; fixed-point theorems. The axiom of choice and Zorn's lemma. Applications of Zorn's lemma in mathematics. The well-ordering principle. [5]

Propositional logic
The propositional calculus. Semantic and syntactic entailment. The deduction and completeness theorems. Applications: compactness and decidability. [3]

Predicate logic
The predicate calculus with equality. Examples of first-order languages and theories. Statement of the completeness theorem; *sketch of proof*. The compactness theorem and the Löwenheim-Skolem theorems. Limitations of first-order logic. Model theory. [5]

Set theory
Set theory as a first-order theory; the axioms of ZF set theory. Transitive closures, epsilon-induction and epsilon-recursion. Well-founded relations. Mostowski's collapsing theorem. The rank function and the von Neumann hierarchy. [5]

Consistency
∗Problems of consistency and independence∗. [1]

Appropriate books

B.A. Davey and H.A. Priestley *Lattices and Order.* Cambridge University Press 2002 (£19.95 paperback).
T. Forster *Logic, Induction and Sets.* Cambridge University Press (£50.00 hardback).
A. Hajnal and P. Hamburger *Set Theory.* LMS Student Texts number 48, CUP 1999 (£55.00 hardback, £22.99 paperback).
A.G. Hamilton *Logic for Mathematicians.* Cambridge University Press 1988 (£25.95 paperback).
† P.T. Johnstone *Notes on Logic and Set Theory.* Cambridge University Press 1987 (£15.95 paperback).
D. van Dalen *Logic and Structure.* Springer-Verlag 1994 (£18.50 paperback).

GRAPH THEORY (D) — 24 lectures, Michaelmas term

No specific prerequisites.

Introduction
Basic definitions. Trees and spanning trees. Bipartite graphs. Euler circuits. Elementary properties of planar graphs. Statement of Kuratowski's theorem. [3]

Connectivity and matchings
Matchings in bipartite graphs; Hall's theorem and its variants. Connectivity and Menger's theorem. [3]

Extremal graph theory
Long paths, long cycles and Hamilton cycles. Complete subgraphs and Turán's theorem. Bipartite subgraphs and the problem of Zarankiewicz. The Erdős-Stone theorem. [5]

Graph colouring
Vertex and edge colourings; simple bounds. The chromatic polynomial. The theorems of Brooks and Vizing. Equivalent forms of the four colour theorem; the five colour theorem. Heawood's theorem for surfaces; the torus and the Klein bottle. [5]

Ramsey theory
Ramsey's theorem (finite and infinite forms). Upper bounds for Ramsey numbers. [3]

Probabilistic methods
Basic notions; lower bounds for Ramsey numbers. The model $\mathcal{G}(n,p)$; graphs of large girth and large chromatic number. The clique number. [3]

Eigenvalue methods
The adjacency matrix and the Laplacian. Strongly regular graphs. [2]

Appropriate books

† B.Bollobás *Modern Graph Theory.* Springer 1998 (£26.00 paperback).
J.A.Bondy and U.S.R.Murty *Graph Theory with Applications.* Elsevier 1976 (available online via http://www.ecp6.jussieu.fr/pageperso/bondy/bondy.html).
R.Diestel *Graph Theory.* Springer 2000 (£38.50 paperback).
D.West *Introduction to Graph Theory.* Prentice Hall 1999 (£45.99 hardback).

GALOIS THEORY (D) — 24 lectures, Michaelmas term

Groups, Rings and Modules is essential.

Field extensions, tower law, algebraic extensions; irreducible polynomials and relation with simple algebraic extensions. Finite multiplicative subgroups of a field are cyclic. Existence and uniqueness of splitting fields. [6]

Existence and uniqness of algebraic closure. [1]

Separability. Theorem of primitive element. Trace and norm. [3]

Normal and Galois extensions, automorphic groups. Fundamental theorem of Galois theory. [3]

Galois theory of finite fields. Reduction mod p. [2]

Cyclotomic polynomials, Kummer theory, cyclic extensions. Symmetric functions. Galois theory of cubics and quartics. [4]

Solubility by radicals. Insolubility of general quintic equations and other classical problems. [3]

Artin's theorem on the subfield fixed by a finite group of automorphisms. Polynomial invariants of a finite group; examples. [2]

Appropriate books

E. Artin *Galois Theory.* Dover Publications (£6.99 paperback).
I. Stewart *Galois Theory.* Taylor & Francis Ltd Chapman & Hall/CRC 3rd edition (£24.99).
B. L. van der Waerden *Modern Algebra.* Ungar Pub 1949 (Out of print).
S. Lang *Algebra (Graduate Texts in Mathematics).* Springer-Verlag New York Inc (£38.50 hardback).
I. Kaplansky *Fields and Rings.* The University of Chicago Press (£16.00 Paperback).

REPRESENTATION THEORY (D) — 24 lectures, Lent term

Linear Algebra, and Groups, Rings and Modules are esssential.

Representations of finite groups
Representations of groups on vector spaces, matrix representations. Equivalence of representations. Invariant subspaces and submodules. Irreducibility and Schur's Lemma. Complete reducibility for finite groups. Irreducible representations of Abelian groups.

Characters
Determination of a representation by its character. The group algebra, conjugacy classes, and orthogonality relations. Regular representation. Induced representations and the Frobenius reciprocity theorem. Mackey's theorem. [12]

Arithmetic properties of characters
Divisibility of the order of the group by the degrees of its irreducible characters. Burnside's p^aq^b theorem. [2]

Tensor products
Tensor products of representations. The character ring. Tensor, symmetric and exterior algebras. [3]

Representations of S^1 and SU_2
The groups S^1 and SU_2, their irreducible representations, complete reducibility. The Clebsch-Gordan formula. *Compact groups.* [4]

Further worked examples
The characters of one of $GL_2(F_q)$, S_n or the Heisenberg group. [3]

Appropriate books

J.L. Alperin and R.B. Bell *Groups and representations.* Springer 1995 (£37.50 paperback).
I.M. Isaacs *Character theory of finite groups.* Dover Publications 1994 (£12.95 paperback).
G.D. James and M.W. Liebeck *Representations and characters of groups.* Second Edition, CUP 2001 (£24.99 paperback).
J-P. Serre *Linear representations of finite groups.* Springer-Verlag 1977 (£42.50 hardback).
M. Artin *Algebra.* Prentice Hall 1991 (£56.99 hardback).

NUMBER FIELDS (D) — 16 lectures, Michaelmas term

Groups, Rings and Modules is essential.

Definition of algebraic number fields, their integers and units. Norms, bases and discriminants. [3]

Ideals, principal and prime ideals, unique factorisation. Norms of ideals. [3]

Minkowski's theorem on convex bodies. Statement of Dirichlet's unit theorem. Determination of units in quadratic fields. [2]

Ideal classes, finiteness of the class group. Calculation of class numbers using statement of the Minkowski bound. [3]

Dedekind's theorem on the factorisation of primes. Application to quadratic fields. [2]

Discussion of the cyclotomic field and the Fermat equation or some other topic chosen by the lecturer. [3]

Appropriate books

† Z.I. Borevich and I.R. Shafarevich *Number Theory.* Elsevier 1986 (out of print).
† J. Esmonde and M.R. Murty *Problems in Algebraic Number Theory.* Springer 1999 (£38.50 hardback).
E. Hecke *Lectures on the Theory of Algebraic Numbers.* Springer 1981 (out of print).
† D.A. Marcus *Number Fields.* Springer 1977 (£30.00 paperback).
I.N. Stewart and D.O. Tall *Algebraic Number Theory and Fermat's Last Theorem.* A K Peters 2002 (£25.50 paperback).

ALGEBRAIC TOPOLOGY (D) — 24 lectures, Michaelmas term

Either Analysis II or Metric and Topological Spaces is essential.

The fundamental group
Homotopy of continuous functions and homotopy equivalence between topological spaces. The fundamental group of a space, homomorphisms induced by maps of spaces, change of base point, invariance under homotopy equivalence. [3]

The fundamental group of the circle
Covering spaces and covering maps. Path-lifting and homotopy-lifting properties, and the fundamental group of the circle. Topological proof of the fundamental theorem of algebra. [3]

Computing the fundamental group
Free groups, generators and relations for groups. Van Kampen's theorem. The fundamental group of the n-sphere, and of the closed (orientable) surface of genus g. [3]

Covering spaces
Statement of the correspondence, for a connected, locally contractible space X, between coverings of X and conjugacy classes of subgroups of the fundamental group of X, *and its proof*. The universal covering of X. The fundamental group of tori and real projective spaces. [3]

Simplicial complexes
Finite simplicial complexes and subdivisions; the simplicial approximation theorem *and its proof*. [2]

Homology
Simplicial homology, the homology groups of a simplex and its boundary. Functorial properties; the invariance of simplicial homology groups under homotopy equivalence. The homology of S^n; Brouwer's fixed-point theorem. [6]

Homology calculations
The Mayer-Vietoris sequence. Determination of the homology groups of closed surfaces. Orientability for surfaces. *The fundamental group and the first homology group.* The Euler characteristic. *Sketch of the classification of closed triangulable surfaces.* [4]

Appropriate books

M. A. Armstrong *Basic topology.* Springer 1983 (£38.50 hardback).
W. Massey *A basic course in algebraic topology.* Springer 1991 (£50.00 hardback).
C. R. F. Maunder *Algebraic Topology.* Dover Publications 1980 (£11.95 paperback).
A. Hatcher *Algebraic Topology.* Cambridge University Press, 2001 (£20.99 paperback).

LINEAR ANALYSIS (D) — 24 lectures, Lent term

Linear Algebra, Analysis II are essential.

Normed and Banach spaces. Linear mappings, continuity, boundedness, and norms. Finite-dimensional normed spaces. [4]

The Baire category theorem. The principle of uniform boundedness, the closed graph theorem and the inversion theorem; other applications. [5]

The normality of compact Hausdorff spaces. Urysohn's lemma and Tiezte's extension theorem. Spaces of continuous functions. The Stone–Weierstrass theorem and applications. Equicontinuity: the Ascoli–Arzelà theorem. [5]

Inner product spaces and Hilbert spaces; examples and elementary properties. Orthonormal systems, and the orthogonalization process. Bessel's inequality, the Parseval equation, and the Riesz–Fischer theorem. Duality; the self duality of Hilbert space. [5]

Bounded linear operations, invariant subspaces, eigenvectors; the spectrum and resolvent set. Compact operators on Hilbert space; discreteness of spectrum. Spectral theorem for compact Hermitian operators. [5]

Appropriate books

† B. Bollobás *Linear Analysis.* 2nd Edition, Cambridge University Press 1999 (£22.99 paperback).
C. Goffman and G. Pedrick *A First Course in Functional Analysis.* 2nd Edition, Oxford University Press 1999 (£21.00 Hardback).
W. Rudin *Real and Complex Analysis.* McGraw–Hill International Editions: Mathematics Series (£31.00 Paperback).

RIEMANN SURFACES (D) — 24 lectures, Lent term

Complex Analysis is essential and Analysis II desirable.

The complex logarithm. Analytic continuation in the plane; natural boundaries of power series. Informal examples of Riemann surfaces of simple functions (via analytic continuation). Examples of Riemann surfaces, including the Riemann sphere, and the torus as a quotient surface. [4]

Analytic, meromorphic and harmonic functions on a Riemann surface; analytic maps between two Riemann surfaces. The open mapping theorem, the local representation of an analytic function as $z \mapsto z^k$. Complex-valued analytic and harmonic functions on a compact surface are constant. [2]

Germs of an analytic map between two Riemann surfaces; the space of germs as a covering surface (in the sense of complex analysis). The monodromy theorem (statement only). The analytic continuation of a germ over a simply connected domain is single-valued. [3]

The degree of a map between compact Riemann surfaces; Branched covering maps and the Riemann-Hurwitz relation (assuming the existence of a triangulation). The fundamental theorem of algebra. Rational functions as meromorphic functions from the sphere to the sphere. [3]

Elliptic functions as functions from a torus to the sphere. Theta functions, and a construction of elliptic functions from theta functions. The Weierstrass $\wp$-function. [4]

Affine and projective algebraic curves; examples of Riemann surfaces of simple algebraic functions. Singular and non-singular points. Discussion of algebraic functions and compact Riemann surfaces. Differentials. The Riemann-Roch theorem for curves (statement only) and some consequences. [6]

A brief discussion of quotient spaces by a discrete group action. The uniformization theorem (statement only). The uniqueness of the conformal structure on the sphere and the non-uniqueness for a torus. The hyperbolic geometry of a Riemann surface. [2]

Appropriate books

L.V.Ahlfors *Complex Analysis.* McGraw-Hill, 1979 (£112.99 Hardback).
A.F.Beardon *A Primer on Riemann Surfaces.* Cambridge University Press, 2004 (£35.00 hardback, £14.95 paperback).
G.A.Jones and D.Singerman *Complex functions: an algebraic and geometric viewpoint.* Cambridge University Press, 1987 (£28.00 Paperback).
F. Kirwan *Complex Algebraic Curves.* Cambridge University Press (£32.50 Paperback).
R. Miranda *Algebraic curves and Riemann surfaces.* American Mathematical Society, 1995 (£30.00 Hardback).
E.T.Whittaker and G.N.Watson *A Course of Modern Analysis Chapters XX and XXI, 4th Edition.* Cambridge University Press, 1996 (£24.99 Paperback).

DIFFERENTIAL GEOMETRY (D) — 24 lectures, Lent term

Analysis II and Geometry are very useful.

Smooth manifolds in $\mathbb{R}^n$, tangent spaces, smooth maps and the inverse function theorem. Examples, regular values, Sard's theorem (statement only). [4]

Notions of transversality and intersection numbers mod 2, degree mod 2. [3]

Curves in 2-space and 3-space, arc-length, curvature, torsion. The isoperimetric inequality. [2]

Smooth surfaces in 3-space, first fundamental form, area. [2]

The Gauss map, second fundamental form, principal curvatures and Gaussian curvature. Theorema Egregium. [3]

Minimal surfaces. Normal variations and characterization of minimal surfaces as critical points of the area functional. Isothermal coordinates and relation with harmonic functions. The Weierstrass representation. Examples. [3]

Parallel transport and geodesics for surfaces in 3-space. Geodesic curvature. [2]

The exponential map and geodesic polar coordinates. The Gauss-Bonnet theorem (including the statement about classification of compact surfaces). [3]

Global theorems on curves: Fenchel's theorem (the total curvature of a simple closed curve is greater than or equal to 2π); the Fary-Milnor theorem (the total curvature of a simple knotted closed curve is greater than 4π). [2]

Appropriate books

M. Do Carmo *Differential Geometry of Curves and Surfaces.* Pearson Higher Education, 1976 (£54.99 hardback).
V. Guillemin and A. Pollack *Differential Topology,.* Pearson Higher Education, 1974 (£54.99 hardback).
J. Milnor *Topology from the differentiable viewpoint.* Revised reprint of the 1965 original. Princeton Landmarks in Mathematics. Princeton University Press, Princeton, NJ, 1997 (£15.95 paperback).
B. O'Neill *Elementary Differential Geometry.* Harcourt 2nd ed 1997 (£58.28 hardback).
A. Pressley *Elementary Differential Geometry,.* Springer Undergraduate Mathematics Series, Springer-Verlag London Ltd, 2000 (£18.95 paperback).
I.M. Singer and J.A. Thorpe *Lecture notes on elementary topology and geometry.* Undergraduate Texts in Mathematics. Springer-Verlag, New York-Heidelberg, 1996 (£46.75 hardback).
M. Spivak *A Comprehensive Introduction to Differential Geometry.* Vols. I-V, Publish or Perish, Inc. 1999 ().
J.A. Thorpe *Elementary Topics in Differential Geometry.* Springer-Verlag 1994 (£33.00 hardback).

PROBABILITY AND MEASURE (D) — 24 lectures, Michaelmas term

Analysis II is essential.

Measure spaces, σ-algebras, π-systems and uniqueness of extension, statement $*$and proof$*$ of Carathéodory's extension theorem. Construction of Lebesgue measure on $\mathbb{R}$. The Borel σ-algebra of $\mathbb{R}$. Existence of non-measurable subsets of $\mathbb{R}$. Lebesgue-Stieltjes measures and probability distribution functions. Independence of events, independence of σ-algebras. The Borel–Cantelli lemmas. Kolmogorov's zero-one law. [6]

Measurable functions, random variables, independence of random variables. Construction of the integral, expectation. Convergence in measure and convergence almost everywhere. Fatou's lemma, monotone and dominated convergence, differentiation under the integral sign. Discussion of product measure and statement of Fubini's theorem. [6]

Chebyshev's inequality, tail estimates. Jensen's inequality. Completeness of L^p for $1 \leqslant p \leqslant \infty$. The Hölder and Minkowski inequalities, uniform integrability. [4]

L^2 as a Hilbert space. Orthogonal projection, relation with elementary conditional probability. Variance and covariance. Gaussian random variables, the multivariate normal distribution. [2]

The strong law of large numbers, proof for independent random variables with bounded fourth moments. Measure preserving transformations, Bernoulli shifts. Statements $*$and proofs$*$ of maximal ergodic theorem and Birkhoff's almost everywhere ergodic theorem, proof of the strong law. [4]

The Fourier transform of a finite measure, characteristic functions, uniqueness and inversion. Weak convergence, statement of Lévy's convergence theorem for characteristic functions. The central limit theorem. [2]

Appropriate books

P. Billingsley *Probability and Measure.* Wiley 1995 (£67.95 hardback).
R.M. Dudley *Real Analysis and Probability.* Cambridge University Press 2002 (£32.95 paperback).
R.T. Durrett *Probability: Theory and Examples.* Wadsworth and Brooks/Cole 1991 (£).
D. Williams *Probability with Martingales.* Cambridge University Press (£20.95 paperback).

APPLIED PROBABILITY (D) — 24 lectures, Lent term

Markov Chains is essential

Finite-state continuous-time Markov chains: basic properties. The homogeneous Poisson process and its generalisations (inhomogeneous Poisson processes, birth processes, birth and death processes, marked Poisson processes) [7]

General continuous-time Markov chains. Jump chains. Communicating classes. Hitting times and probabilities. Recurrence and transience. Positive and null recurrence. Convergence to equilibrium. Reversibility. [5]

Applications: the $M/M/1$ queue, the loss system, the $M/M/\infty$ system, the $M/M/s$ queue distribution of impurities, ∗Bartlett's theorem∗, the $M/G/\infty$ queue, shot-noise processes, line processes. [5]

Renewal and renewal-reward processes. Applications: occurrence of patterns, maintenance models, regenerative processes, Little's formula, the $M/G/1$ queue, reservoir, epidemic and insurance ruin models. [7]

Appropriate books

J.F.C. Kingman *Poisson Processes.* OUP 1993 (£30.00 hardback).
G.R. Grimmett and D.R. Stirzaker *Probability and Random Processes.* OUP 2001 (£29.95 paperback).
J.R. Norris *Markov Chains.* CUP 1997 (£20.95 paperback).
S.M. Ross *Stochastic Processes.* Wiley 1996 (£17.95 paperback).

PRINCIPLES OF STATISTICS (D) — 24 lectures, Michaelmas term

Statistics is essential

Decision theory
Basic elements of a decision problem: sample, parameter and action spaces, loss functions, risk. Decision rules, admissibility, randomised decision rules. Decision principles: Bayes and minimax principles. Risk set. Extended Bayes rules, equalizer rules. [3]

Bayesian inference
Prior and posterior distributions. Conjugate families, improper priors, predictive distributions. Bayesian inference: estimation, confidence regions, hypothesis testing. Bayes factors, model selection. Philosophy of Bayesian inference. [3]

The general form of Bayes rules. Admissibility of Bayes rules. Empirical Bayes, shrinkage. [2]

Bayesian computation via Monte Carlo simulation: algorithms and examples. [1]

Classical inference
Data reduction. Sufficiency, minimal sufficiency, completeness. Exponential families. [2]

Optimal point estimation. Cramer-Rao lower bound, Rao-Blackwell theorem, Lehmann-Scheffe theorem. [2]

Optimal hypothesis testing. Monotone likelihood ratio, uniformly most powerful tests. Unbiased tests. Uniformly most powerful unbiased tests in one-parameter exponential families. Nuisance parameters. Conditional tests. The conditionality principle. Conditional tests in multi-parameter exponential families. [5]

Likelihood theory. Maximum likelihood estimates and their asymptotic properties: consistency, efficiency and normality. Wald, score and likelihood ratio tests. Generalised likelihood ratio tests: applications to generalised linear models. Multiparameter problems. Profile likelihood. [4]

Bootstrap inference: bias, variance, distribution and confidence set estimation. [2]

Appropriate books

J.O. Berger *Statistical Decision Theory and Bayesian Analysis.* Springer 1985 (£41.50 hardback).
D.R. Cox and D.V. Hinkley *Theoretical Statistics.* Chapman and Hall/CRC 1974 (£39.99 paperback).
A.C. Davison *Statistical Models.* Cambridge University Press 2003 (£40.00 hardback).
E.L. Lehmann and G. Casella *Theory of Point Estimation.* Springer 1998 (£59.00 hardback).
E.L. Lehmann *Testing Statistical Hypotheses.* Springer 1997 (£49.00 hardback).
J. Shao and D. Tu *The Jackknife and Bootstrap.* Springer 1995 (£35.50 hardback).

STOCHASTIC FINANCIAL MODELS (D) — 24 lectures, Lent term

Methods, Statistics and Markov Chains are desirable.

Utility and mean-variance analysis
Utility functions; risk aversion and risk neutrality. Portfolio selection with the mean-variance criterion; the efficient frontier when all assets are risky and when there is one riskless asset. The capital-asset pricing model. Reservation bid and ask prices, marginal utility pricing. Simplest ideas of equilibrium and market clearning. State-price density. [5]

Martingales
Conditional expectation, definition and basic properties. Conditional expectation, definition and basic properties. Stopping times. Martingales, supermartingales, submartingales. Use of the optional sampling theorem. [3]

Dynamic models
Introduction to dynamic programming; optimal stopping and exercising American puts; optimal portfolio selection. [3]

Pricing contingent claims
Lack of arbitrage in one-period models; hedging portfolios; martingale probabilities and pricing claims in the binomial model. Extension to the multi-period binomial model. Axiomatic derivation. [4]

Brownian motion
Introduction to Brownian motion; Brownian motion as a limit of random walks. Hitting-time distributions; changes of probability. [3]

Black-Scholes model
The Black-Scholes formula for the price of a European call; sensitivity of price with respect to the parameters; implied volatility; pricing other claims. Binomial approximation to Black-Scholes. Use of finite-difference schemes to computer prices [6]

Appropriate books

J. Hull *Options, Futures and Other Derivative Securities.* Prentice-Hall 2003 (£42.99 paperback).
J. Ingersoll *Theory of Financial Decision Making.* Rowman and Littlefield 1987 (£52.00 hardback).
A. Rennie and M. Baxter *Financial Calculus: an introduction to derivative pricing.* Cambridge University Press 1996 (£35.00 hardback).
P. Wilmott, S. Howison and J. Dewynne *The Mathematics of Financial Derivatives: a student introduction.* Cambridge University Press 1995 (£22.95 paperback).

OPTIMIZATION AND CONTROL (D) — 16 lectures, Michaelmas term

Optimization and Markov Chains are very helpful.

Dynamic programming
The principle of optimality. The dynamic programming equation for finite-horizon problems. Interchange arguments. Markov decision processes in discrete time. Infinite-horizon problems: positive, negative and discounted cases. Value interation. Policy improvment algorithm. Stopping problems. Average-cost programming. [6]

LQG systems
Linear dynamics, quadratic costs, Gaussian noise. The Riccati recursion. Controllability. Stabilizability. Infinite-horizon LQ regulation. Observability. Imperfect state observation and the Kalman filter. Certainty equivalence control. [5]

Continuous-time models
The optimality equation in continuous time. Pontryagin's maximum principle. Heuristic proof and connection with Lagrangian methods. Transversality conditions. Optimality equations for Markov jump processes and diffusion processes. [5]

Appropriate books

D.P. Bertsekas *Dynamic Programming and Optimal Control, Volumes I and II.* Athena Scientific, 2001 ($109.50).
L. M. Hocking *Optimal Control: An introduction to the theory with applications.* Oxford 1991 (£27.99).
S. Ross *Introduction to Stochastic Dynamic Programming.* Academic Press 1995 (£27.00 paperback).
P. Whittle *Optimization over Time Vols. I and II.* Wiley 1983 (out of print).
P. Whittle *Optimal Control: Basics and Beyond.* Wiley 1995 (£130).

PARTIAL DIFFERENTIAL EQUATIONS (D) — 24 lectures, Michaelmas term

Analysis II, Methods and either of Complex Methods or Complex Analysis are essential and Metric and Topological Spaces is desirable.

Introduction

The concept of a partial differential equation. Examples of linear and nonlinear equations. Discussion of solvability and well-posedness. Relevance to mathematical physics: Laplace, diffusion and wave equations. Linear differential operators and their total and principal symbol. The characteristic set, elliptic, parabolic and hyperbolic equations. [2]

First-order equations

Vector fields and flows on domains in $\mathbb{R}^n$. Critical points, relation to dynamical systems. Quasilinear equations. The solution of first-order quasilinear hyperbolic equations by characteristics. [3]

Fundamental solutions and distributions

Brief overview of Schwartz functions and tempered distributions. The Fourier transform; statement *and proof* of inversion formula; convolution. Fundamental solutions of equations with constant coefficients. Brief discussion of connection with Green functions and resolvent kernels. Statement of existence theorem for fundamental solutions with discussion. [5]

The Laplace equation

Fundamental solution. Solution by separation of variables. Cauchy–Riemann equations *and solution in general simply-connected domains by means of a conformal mapping*. The Poisson equation; statement of elliptic regularity. Maximum principle. Mean value property. The concept of a weak solution and reduction of a Poisson equation to a variational problem. Proof of the existence and uniqueness of weak solutions for the Poisson equation using variational formulation. [7]

The diffusion equation

Fundamental solution. Maximum principle. Solution of the inhomogeneous equation. [3]

The wave equation

Fundamental solution in one, two and three spatial dimensions. Method of descent. Solution of the Cauchy initial value problem for $t = 0$. Finite propagation speed. The Huygens principle. [4]

Appropriate books

L.C. Evans *Partial Differential equations.*. OUP 1998 (£?).
F.G. Friedlander and M. Joshi *Introduction to the Theory of Distributions.* CUP 1999 (£50.00 hardback, £18.99 paperback).
G. Folland *Introduction to Partial Differential Equations.* Princeton University Press 1996 (£42.84 hardback).
F. John *Partial Differential Equations.* Springer 1995 (£46.75 hardback).

ASYMPTOTIC METHODS (D) — 16 lectures, Michaelmas term

Either Complex Methods or Complex Analysis is essential, Part II Further Complex Methods is useful.

Asymptotic expansions

Definition (Poincare) of $\phi(z) \sim \sum a_n z^{-n}$; examples; elementary properties; uniqueness; Stokes' phenomenon. [4]

Asymptotics behaviour of functions defined by integrals

Integration by parts. Watson's lemma (including proof) and Laplace's method. Riemann–Lebesgue lemma (including proof) and method of stationary phase. The method of steepest descent (including derivation of higher order terms). Application to Airy function. [7]

Asymptotic behaviour of solutions of differential equations

Asymptotic solution of second-order linear differential equations, including Liouville–Green functions (proof that they are asymptotic not required) and WKBJ with the quantum harmonic oscillator as an example. [4]

Recent developments

Further discussion of Stokes' phenomenon. *Asymptotics 'beyond all orders'*. [1]

Appropriate books

† M.J. Ablowitz and A.S. Fokas *Complex Variables: Introduction and Applications.* CUP 2003 (£32, paperback).
J.D. Murray *Asymptotic Analysis.* Springer 1984 (£41.00 hardback).
A. Erdelyi *Asymptotic Expansions.* Dover 1956 (£6.95 paperback).
F.W.J. Olver *Asymptotics and Special Functions.* A K Peters 1997 (£46.50 hardback).

INTEGRABLE SYSTEMS (D) — 24 lectures, Lent term

Methods, and Complex Methods or Complex Analysis are essential, Part II Further Complex Methods is desirable.

Introduction

The rich mathematical structure and the genericity of the standard integrable nonlinear partial differential equations (Korteweg-de Vries, nonlinear Schrödinger, sine-Gordon). [1]

Burgers equation

The Cole-Hopf transformation, physical applications including models arising in fluid mechanics. [2]

Introduction to the Riemann-Hilbert problem and to singular integral equations

Cauchy type integrals, scalar Riemann-Hilbert problems, Singular Integral equations. [3]

Direct methods and soliton solutions

Bäcklund transformations, the Hirota method, the direct linearising method, the Dressing method. [3]

The inverse scattering (spectral) method

Lax pairs for linear PDEs, the inverse scattering method for linear PDEs, the inverse scattering mehod for the KdV equation. [4]

Painlevé equations

The classical Painlevé equations, the ODE reductions of certain integrable nonlinear PDEs, introduction to the isomonodromy methods. [3]

Appropriate books

MJ Ablowitz and P Clarkson *Solitons, Nonlinear Evolution Equations and Inverse Scattering.* CUP, 1991 (£50.00 paperback).

MJ Ablowitz and AS Fokas *Complex Variables, Second Edition.* CUP, 2003 (£32.00 paperback).

PRINCIPLES OF QUANTUM MECHANICS (D) — 24 lectures, Michaelmas term

Quantum Mechanics is essential.

Dirac formalism

Bra and ket notation, operators and observables, probability amplitudes, expectation values, complete commuting sets of operators, unitary operators. Schrödinger equation, wave functions in position and momentum space. [3]

Time evolution operator, Schrödinger and Heisenberg pictures, Heisenberg equations of motion. [2]

Harmonic oscillator

Analysis using annihilation, creation and number operators. Significance for normal modes in physical examples. [2]

Multiparticle systems

Composite systems and tensor products, wave functions for multiparticle systems. Symmetry or anti-symmetry of states for identical particles, Bose and Fermi statistics, Pauli exclusion principle. [3]

Perturbation theory

Time-independent theory; second order without degeneracy, first order with degeneracy. [2]

Angular momentum

Analysis of states $|jm\rangle$ from commutation relations. Addition of angular momenta, calculation of Clebsch–Gordan coefficients. Spin, Pauli matrices, singlet and triplet combinations for two spin half states. [4]

Translations and rotations

Unitary operators corresponding to spatial translations, momenta as generators, conservation of momentum and translational invariance. Corresponding discussion for rotations. Reflections, parity, intrinsic parity. [3]

Time-dependent perturbation theory

Interaction picture. First-order transition probability, the golden rule for transition rates. Application to atomic transitions, selection rules based on angular momentum and parity, *absorption, stimulated and spontaneous emission of photons*. [3]

Quantum basics

Quantum data, qubits, no cloning theorem. Entanglement, pure and mixed states, density matrix. Classical determinism versus quantum probability, Bell inequality for singlet two-electron state, GHZ state. [2]

Appropriate books

† E. Merzbacher *Quantum Mechanics, 3rd edition.* Wiley 1998 (£36.99 hardback).

† B.H. Bransden and C.J. Joachaim *Quantum Mechanics, 2nd edition.* Pearson (£37.99 paperback).

J.J. Sakurai *Modern Quantum Mechanics, 2nd edition.* Addison Wesley 1993 (£66.99 hardback).

P.A.M. Dirac *The Principles of Quantum Mechanics.* Oxford University Press 1967, reprinted 2003 (£24.99 paperback).

C.J. Isham *Lectures on Quantum Theory: Mathematical and Structural Foundations.* Imperial College Press 1995 (£14.00 paperback).

APPLICATIONS OF QUANTUM MECHANICS (D) — 24 lectures, Lent term

Principles of Quantum Mechanics is essential.

Scattering theory

Scattering and definition of differential cross-section. Asymptotic wave function for quantum scattering, scattering amplitude; Green functions; Born approximation to scattering on a potential. Spherically symmetric potential, phase shifts, optical theorem. Low energy scattering; bound states and resonances as poles in scattering amplitude. Elastic and inelastic scattering on a bound state, Born approximation. [7]

Atomic physics

Variational principle. Helium atom. Periodic Table. Binding of H_2^+. [3]

Solid state physics

Crystal structure, reciprocal lattice. Body-centred and face-centred cubic lattices. Bragg scattering on a rigid lattice. [2]

Band theory

Bloch's theorem. Band structure, nearly free electron and tight-binding models, exact solution with one-dimensional δ-function potentials. Fermi surface. [3]

Dynamics of electrons in a crystal under an applied electric field. Conductors and insulators. [1]

Statement of Fermi-Dirac distribution, chemical potential. Semiconductors. Holes as current carriers. Doping, *pn*-junction and its applications. [3]

Phonons

Elastic vibrations, phonons. Born approximation to neutron scattering in a crystal, effect of zero point motion on elastic and inelastic scattering. [3]

Electrons in magnetic fields

Landau levels. The de Haas–van Alphen effect. [2]

Appropriate books

E. Merzbacher *Quantum Mechanics.* 3rd edition, Wiley 1998 (£36.99 hardback).
N.W. Ashcroft and N.D. Mermin *Solid State Physics.* Holt–Saunders 1976 (£36.99 hardback).
J.M. Ziman *Principles of the Theory of Solids.* 2nd edition, Cambridge University Press 1979 (£32.95 paperback).

STATISTICAL PHYSICS (D) — 16 lectures, Lent term

Quantum Mechanics is essential and Part II Classical Dynamics is desirable.

Thermal equilibrium. Thermal isolation. Micro- and macro-states. Microcanonical distribution. Definition of entropy, temperature. Adiabatic changes, pressure. [2]

First and second laws of thermodynamics. $dE = TdS - PdV$. Definition of free energy. Thermodynamic formulae, including Maxwell relations, specific heats. Equations of state. Example of perfect gas. [2]

Canonical (Gibbs) distribution for a quantum system. Partition function and free energy. Fluctuations. [1]

Classical Boltzmann distribution. Ideal monatomic classical gas. Maxwell distribution. Equipartition theorem. Entropy. Diatomic ideal gas. *Increase in entropy for an isolated system: e.g. mixing of ideal gases.* Simple theory of paramagnetism. Mention of effect of inter-molecular forces, treated by perturbation theory. [4]

Classical and quantum simple harmonic oscillator. [1]

Variable particle number, grand thermodynamic potential. Chemical potential. (Grand) Gibbs distribution and partition function. Bose and Fermi distribution functions. Fluctuations. Black-body radiation. Planck distribution and Stefan law. Brief account of Debye model for heat capacity of solid. [3]

Ideal quantum monatomic gas. Correction to classical equation of state at low density. Behaviour of chemical potential as a function of temperature. Degenerate free electron gas. Electronic specific heat at low temperature. Bose condensation. [3]

Appropriate books

R.K. Pathria *Statistical Mechanics, 2nd edition.* Butterworth-Heinemann 1996 (£41.99 paperback).
L.D. Landau and E.M. Lifshitz *Statistical Physics, Part 1 (Course of Theoretical Physics volume 5).* Butterworth-Heinemann 1996 (£36.99 paperback).
M. Toda, R. Kubo and N. Saito *Statistical Physics I.* Springer-Verlag 1992 (£38.50 paperback).
K. Huang *Introduction to Statistical Physics.* Taylor and Francis 2001 (£24.99 hardback).
F. Mandl *Statistical Physics.* Wiley 1988 (£27.50 paperback).

ELECTRODYNAMICS (D) — 16 lectures, Michaelmas term

Special Relativity and Electromagnetism are essential and Methods is desirable.

Electrostatics
Revision of electrostatics. Potential due to a localized charge distribution. Dipole and quadrupole moments. [2]

Maxwell's equations
Relativistic form of Maxwell's equations and Lorentz force law. 4-vector potential and gauge invariance. Lorentz transformations. [3]

Charged particle dynamics
4-momentum and 4-force. Motion of charged particles in constant electric and magnetic fields. [2]

Energy and momentum
Electromagnetic field energy, momentum and stress tensor. Poynting's theorem. Energy and momentum density of a plane electromagnetic wave. Radiation pressure. [2]

Electromagnetic radiation
Retarded potential of a time-dependent charge distribution. The radiation field. Dipole radiation. Energy radiated. [3]

Variational principles
Lagrangian for electromagnetic field with prescribed current, and for charged particles. [2]

Introduction to superconductors
Scalar electrodynamics as a model for a superconductor. Ginzburg–Landau Lagrangian and field equations. Magnetic flux and its quantization. [2]

Appropriate books

R. Feynman, R. Leighton and M. Sands *The Feynman Lectures in Physics Vol.2 and Vol.3 Chap. 21.* Addison–Wesley 1970 (£87.99 paperback).

† J.D. Jackson *Classical Electrodynamics.* Wiley 1999 (£63.95 hardback).

L. Landau and E.M. Lifshitz *The Classical Theory of Fields.* Butterworth-Heinemann 1975 (£34.99 paperback).

E.M. Lifshitz and L.P. Pitaevskii *Statistical Physics Part 2: Theory of the condensed state.* Butterworth-Heinemann 1980 (£34.99 paperback).

M. Schwartz *Principles of Electrodynamics.* Dover 1987 (£14.95 paperback).

J. Schwinger, et al. *Classical Electrodynamics.* Perseus 1998 (£56.50 hardback).

GENERAL RELATIVITY (D) — 16 lectures, Lent term

Methods and Special Relativity are essential.

Curved and Riemannian spaces. Special relativity and gravitation, the Pound–Rebka experiment. Introduction to general relativity: interpretation of the metric, clock hypothesis, geodesics, equivalence principles. Static spacetimes, Newtonian limit. [4]

Covariant and contravariant tensors, tensor manipulation, partial derivatives of tensors. Metric tensor, magnitudes, angles, duration of curve, geodesics. Connection, Christoffel symbols, absolute and covariant derivatives, parallel transport, autoparallels as geodesics. Curvature. Riemann and Ricci tensors, geodesic deviation. [5]

Vacuum field equations. Spherically symmetric spacetimes, the Schwarzschild solution. Rays and orbits, gravitational red-shift, light deflection, perihelion advance. Event horizon, gravitational collapse, black holes. [5]

Equivalence principles, minimal coupling, non-localisability of gravitational field energy. Bianchi identities. Field equations in the presence of matter, equations of motion. [2]

Appropriate books

J.B. Hartle *Gravity: An introduction to Einstein's General Relativity.* Addison–Wesley 2002 (£32.99 hardback).

L.P. Hughston and K.P. Tod *An Introduction to General Relativity.* Cambridge University Press 1990 (£16.95 paperback).

† R. d'Inverno *Introducing Einstein's Relativity.* Clarendon 1992 (£29.95 paperback).

W. Rindler *Relativity: Special, General and Cosmological.* Oxford University Press 2001 (£24.95 paperback).

B.F. Schutz *A First Course in General Relativity.* Cambridge University Press 1985 (£24.95 paperback).

H. Stephani *General Relativity: an Introduction to the Theory of the Gravitational Field.* Cambridge University Press 1990 (£25.95 paperback).

H. Stephani *Relativity: An introduction to Special and General Relativity.* Cambridge University Press, 2004 (£30.00 paperback, £80.00 hardback).

N. Straumann *General Relativity: with applications to astrophysics.* Springer 2004 (£54.00 hardback).

FLUID DYNAMICS II (D) — 24 lectures, Michaelmas term

Methods and Fluid Dynamics are essential.

Governing equations for an incompressible Newtonian fluid
Stress and rate-of-strain tensors and hypothesis of linear relation between them for an isotropic fluid; equation of motion; conditions at a material boundary; dissipation; flux of mass, momentum and energy; the Navier-Stokes equations. Dynamical similarity; steady and unsteady Reynolds numbers. [4]

Unidirectional flows
Couette and Poiseuille flows; the Stokes layer; the Rayleigh problem. [2]

Stokes flows
Flow at low Reynolds number; linearity and reversibility; uniqueness and minimum dissipation theorems. Flow in a corner; force and torque relations for a rigid particle in arbitrary motion; case of a rigid sphere and a spherical bubble. [4]

Flow in a thin layer
Lubrication theory; simple examples; the Hele-Shaw cell; gravitational spreading on a horizontal surface. [3]

Generation and confinement of vorticity
Vorticity equation; vortex stretching; flow along a plane wall with suction; flow toward a stagnation point on a wall; flow in a stretched line vortex. [3]

Boundary layers at high Reynolds number
The Euler limit and the Prandtl limit; the boundary layer equation for two-dimensional flow. Similarity solutions for flow past a flat plate and a wedge. ∗Discussion of the effect of acceleration of the external stream, separation.∗ Boundary layer at a free surface; rise velocity of a spherical bubble. [6]

Stability of unidirectional inviscid flow
Instability of a vortex sheet and of simple jets (e.g. vortex sheet jets). [2]

Appropriate books

D.J. Acheson *Elementary Fluid Dynamics.* Oxford University Press 1990 (£23.50 paperback).
G.K. Batchelor *An Introduction to Fluid Dynamics.* Cambridge University Press 2000 (£19.95 paperback).
E. Guyon, J-P Hulin, L. Petit and C.D. Mitescu *Physical Hydrodynamics.* Oxford University Press 2000 (£59.50 hardback; £29.50 paperback).
L.D. Landau and E.M. Lifshitz *Fluid Mechanics.* Butterworth-Heinemann 1987 (£38.99 paperback).

WAVES (D) — 24 lectures, Lent term

Methods is essential and Fluid Dynamics is very helpful.

Sound waves
Equations of motion of an inviscid compressible fluid (without discussion of thermodynamics). Mach number. Linear acoustic waves; wave equation; wave-energy equation; plane waves. [3]

Elastic waves
Momentum balance; stress and infinitesimal strain tensors and hypothesis of a linear relation between them for an isotropic solid. Wave equations for dilatation and rotation; dilatation and shear potentials. Compressional and shear plane waves; simple problems of reflection and transmission. [6]

Dispersive waves
Rectangular acoustic wave guide; Love waves; cut-off frequency. Representation of a localised initial disturbance by a Fourier integral (one-dimensional case only); modulated wave trains; stationary phase. Group velocity as energy propagation velocity; dispersing wave trains. Water waves; internal gravity waves. [5]

Ray theory
Group velocity from wave-crest kinematics; ray tracing equations. Doppler effect; ship wave pattern. Cases where Fermat's Principle and Snell's law apply. [4]

Non-linear waves
One-dimensional unsteady flow of a perfect gas. Water waves. Riemann invariants; development of shocks; rarefaction waves; 'piston' problems. Rankine-Hugoniot relations for a steady shock. Hydraulic jumps. [6]

Appropriate books

J.D. Achenbach *Wave Propagation in Elastic Solids.* North Holland 1973 (approx. £60.00).
† J. Billingham and A.C. King *Wave Motion: Theory and application.* Cambridge University Press 2000 (£70.00 hardback, £24.95 paperback).
D.R. Bland *Wave Theory and Applications.* Clarendon Press 1988 (out of print).
M.J. Lighthill *Waves in Fluids.* Cambridge University Press 1978 (£36.95 paperback).
G.B. Whitham *Linear and Nonlinear Waves.* Wiley 1999 (£53.95 paperback).

NUMERICAL ANALYSIS (D) — 24 lectures, Lent term

Part IB Numerical Analysis is essential and Analysis II, Linear Algebra and Complex Methods or Complex Analysis are all desirable.

Iterative methods for linear algebraic systems

Standard methods revisited: Jacobi and Gauss–Seidel. Krylov spaces. Conjugate gradients and pre-conditioning. [3]

Eigenvalues and eigenvectors

The power method and inverse iteration. Deflation. Transformations to tridiagonal and upper Hessenberg forms. The QR algorithm for symmetric and general matrices, including shifts. [5]

Ordinary differential equations

Euler's method and proof of convergence. Multistep methods, including order, the root condition and the concept of convergence. Runge–Kutta schemes. Stiff equations and A-stability. Techniques for error control. [6]

Poisson's equation

Approximation of ∇^2 by finite differences. The accuracy of the five-point method in a square. Solution of the difference equations by iterative methods, including multigrid. Fast Fourier transform techniques. [5]

Finite difference methods for initial value partial differential equations

Difference schemes for the diffusion equation and the advection equation. Proof of convergence in simple cases. Stability analysis by Fourier techniques. Splitting methods. [5]

Appropriate books

G.H. Golub and C.F. van Loan *Matrix Computations.* Johns Hopkins Press 1996 (out of print).

A. Iserles *A First Course in the Numerical Analysis of Differential Equations.* Cambridge University Press 1996 (£23.95 paperback).

J.D. Lambert *Numerical Methods for Ordinary Differential Systems.* Wiley 1991 (£39.95 hardback).

G.D. Smith *Numerical Solution of Partial Differential Equations: Finite Difference Methods.* Oxford University Press 1985 (£27.50 paperback).

COMPUTATIONAL PROJECTS — 6 lectures (Michaelmas) plus practical work

The Faculty publishes a projects booklet by the end of the month of July preceding the Part II year. This contains details of the projects and information about course administration. The booklet is available on the Faculty website at `http://www.maths.cam.ac.uk/catam/`

Each project is allocated a number of units of credit. Full credit nmay obtained from the submission of projects with credit totalling 30 units. Credit for submissions totalling less than 30 units is awarded proportionately. There is no restriction on the choice of projects.

Once the booklet is available, the projects may be done at any time up to the submission deadline, which is in the near the beginning of the Easter Full Term.

The lectures will cover some or all of the topics listed below.

Introduction

Administrative arrangements and overall review of projects. How to write up a project; sample illustrative write-up. [2]

Applied mathematics projects

Review of projects and software packages. Brief introduction to numerical methods for partial differential equations and discrete Fourier transforms. [2]

Pure mathematics projects

Review of projects and software packages. [1]

Probability and statistics projects

Review of projects and software packages. [1]

Appropriate books

† Faculty of Mathematics, University of Cambridge *The CATAM Software Library.* Available from CMS Reception (about £4.00).

† R.L. Burden and J.D. Faires *Numerical Analysis.* Brooks Cole 7th ed. 2001 (£33.00 hardback).

S.D. Conte and C. de Boor *Elementary Numerical Analysis.* McGraw–Hill 1980 (£93.99 hardback; £31.99 paperback).

C.F. Gerald and P.O. Wheatley *Applied Numerical Analysis* 6th ed.. Addison–Wesley 1999 (£32.99 paperback; £39.99 hardback).

W.H. Press, B.P. Flannery, S.A. Teukolsky and W.T. Vetterling *Numerical Recipes: the Art of Scientific Computing* (various editions based on different computer languages). Cambridge University Press 1993 (£42.50 hardback).