

Part II — Statistical Physics

Definitions

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These notes are not endorsed by the lecturers, and I have modified them (often significantly) after lectures. They are nowhere near accurate representations of what was actually lectured, and in particular, all errors are almost surely mine.

Part IB Quantum Mechanics and “Multiparticle Systems” from Part II Principles of Quantum Mechanics are essential

Fundamentals of statistical mechanics

Microcanonical ensemble. Entropy, temperature and pressure. Laws of thermodynamics. Example of paramagnetism. Boltzmann distribution and canonical ensemble. Partition function. Free energy. Specific heats. Chemical Potential. Grand Canonical Ensemble. [5]

Classical gases

Density of states and the classical limit. Ideal gas. Maxwell distribution. Equipartition of energy. Diatomic gas. Interacting gases. Virial expansion. Van der Waal’s equation of state. Basic kinetic theory. [3]

Quantum gases

Density of states. Planck distribution and black body radiation. Debye model of phonons in solids. Bose–Einstein distribution. Ideal Bose gas and Bose–Einstein condensation. Fermi–Dirac distribution. Ideal Fermi gas. Pauli paramagnetism. [8]

Thermodynamics

Thermodynamic temperature scale. Heat and work. Carnot cycle. Applications of laws of thermodynamics. Thermodynamic potentials. Maxwell relations. [4]

Phase transitions

Liquid-gas transitions. Critical point and critical exponents. Ising model. Mean field theory. First and second order phase transitions. Symmetries and order parameters. [4]

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0 Introduction

1 Fundamentals of statistical mechanics

1.1 Microcanonical ensemble

Definition (Microstate). The *microstate* of a system is the actual (quantum) state of the system. This gives a complete description of the system.

Definition (Expectation value). Given a probability distribution $p(n)$ on the states, the expectation value of an operator \mathcal{O} is

$$\langle \mathcal{O} \rangle = \sum_n p(n) \langle n | \mathcal{O} | n \rangle.$$

Definition (Microcanonical ensemble). In a *microcanonical ensemble*, we know the energy is between E and $E + \delta E$, where δE is the accuracy of our measuring device. The accessible microstates are those with energy $E \leq E_n \leq E + \delta E$. We let $\Omega(E)$ be the number of such states.

Definition (Boltzmann entropy). The (*Boltzmann*) *entropy* is defined as

$$S(E) = k \log \Omega(E),$$

where $k = 1.381 \times 10^{-23} \text{ J K}^{-1}$ is *Boltzmann's constant*.

Definition (Temperature). The *temperature* is defined to be

$$\frac{1}{T} = \frac{dS}{dE}.$$

Definition (Heat capacity). The *heat capacity* of a system is

$$C = \frac{dE}{dT}.$$

The *specific heat capacity* is

$$\frac{C}{\text{mass of system}}.$$

1.2 Pressure, volume and the first law of thermodynamics

Definition (Temperature). The *temperature* of a system with variable volume is

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E} \right)_V,$$

with V fixed.

Definition (Pressure). We define the *pressure* of a system with variable volume to be

$$p = T \left(\frac{\partial S}{\partial V} \right)_E.$$

1.3 The canonical ensemble

Notation (β).

$$\beta = \frac{1}{kT}.$$

Definition (Partition function). The *partition function* is

$$Z = \sum_n e^{-\beta E_n}.$$

Definition (Gibbs entropy). The *Gibbs entropy* of a probability distribution $p(n)$ is

$$S = -k \sum_n p(n) \log p(n).$$

1.4 Helmholtz free energy

Definition (Helmholtz free energy). The *Helmholtz free energy* is

$$F = \langle E \rangle - TS.$$

1.5 The chemical potential and the grand canonical ensemble

Definition (Chemical potential). The *chemical potential* of a system is given by

$$\mu = -T \left(\frac{\partial S}{\partial N} \right)_{E,V}.$$

Definition (Grand canonical potential). The *grand canonical potential* is

$$\Phi = F - \mu N = E - TS - \mu N.$$

1.6 Extensive and intensive properties

Definition (Extensive quantity). An *extensive quantity* is one that scales proportionally to the size of the system.

Definition (Intensive quantity). An *intensive quantity* is one that is independent of the size of the system.

2 Classical gases

2.1 The classical partition function

Definition (Partition function (single particle)). We define the *single particle partition function* as

$$Z_1 = \frac{1}{h^3} \int d^3q d^3p e^{-\beta H(\mathbf{p}, \mathbf{q})}.$$

2.2 Monoatomic ideal gas

Definition (Ideal gas). An *ideal gas* is a gas where the particles do not interact with each other.

Definition (Thermal de Broglie wavelength). The *thermal de Broglie wavelength* of a gas at temperature T is

$$\lambda = \sqrt{\frac{2\pi\hbar^2}{mkT}}.$$

Definition (Equation of state). An *equation of state* is an equation that relates state variables, i.e. variables that depend only on the current state of the system, as opposed to how we obtained this system.

2.3 Maxwell distribution

2.4 Diatomic gases

2.5 Interacting gases

3 Quantum gases

3.1 Density of states

3.2 Black-body radiation

3.3 Phonons and the Debye model

3.4 Quantum ideal gas

3.5 Bosons

3.6 Bose–Einstein condensation

3.7 Fermions

Definition (Fermi energy). The *Fermi energy* is

$$E_f = \mu(T = 0) = \lim_{T \rightarrow 0} \mu(T, V, N).$$

3.8 Pauli paramagnetism

Definition (Magnetization). The *magnetization* is

$$M = - \left(\frac{\partial E}{\partial B} \right)_{S, V, N}.$$

Definition (Magnetic susceptibility). The *magnetic susceptibility* is

$$\chi = \left(\frac{\partial M}{\partial \beta} \right)_{T, V, N}.$$

4 Classical thermodynamics

4.1 Zeroth and first law

Definition (Wall). A *wall* is a rigid boundary that matter cannot cross.

Definition (Adiabatic wall). *Adiabatic walls* isolate the system completely from external influences, i.e. the system is *insulated*.

Definition (Diathermal wall). A non-adiabatic wall is called *diathermal*. Systems separated by a diathermal wall are said to be in *thermal contact*.

Definition (Equilibrium). An isolated system with a time-independent state is said to be in *equilibrium*.

Two systems are said to be in equilibrium if when they are put in thermal contact, then the whole system is in equilibrium.

Definition (Temperature). *Temperature* is an equivalence class of systems with respect to the “equilibrium” relation.

Definition (Quasi-static change). A change is *quasi-static* if it is done so slowly that the system remains in equilibrium throughout the change.

Definition (Reversible change). A change is *reversible* if the time-reversal process is possible.

4.2 The second law

4.3 Carnot cycles

Definition (Efficiency). The *efficiency* of a heat engine is

$$\eta = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}.$$

4.4 Entropy

Definition (Entropy). The *entropy* of a system at $A = (p, V)$ is given by

$$S(A) = \int_0^A \frac{dQ}{T},$$

where 0 is some fixed reference state, and the integral is evaluated along any reversible path.

4.5 Thermodynamic potentials

Definition (Ideal gas). An *ideal gas* is a gas that satisfies *Boyle’s law*, which says pV is just a function of T , say $pV = f(T)$, and *Joule’s law*, which says E is a function of T .

4.6 Third law of thermodynamics

5 Phase transitions

5.1 Liquid-gas transition

5.2 Critical point and critical exponents

Definition (First-order phase transition). A *first-order* phase transition is one with a discontinuity in a first derivative of G (or F).

Definition (Second-order phase transition). A *second-order phase transition* is one with continuous first order derivatives, but some second (or higher) derivative of G (or F) exhibits some kind of singularity.

5.3 The Ising model

5.4 Landau theory

Definition (Correlation function). We define the *correlation function*

$$G_{ij} = \langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle.$$