Part III — Hydrodynamic Stability

Theorems with proof

Based on lectures by C. P. Caulfield

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Developing an understanding by which "small" perturbations grow, saturate and modify fluid flows is central to addressing many challenges of interest in fluid mechanics. Furthermore, many applied mathematical tools of much broader relevance have been developed to solve hydrodynamic stability problems, and hydrodynamic stability theory remains an exceptionally active area of research, with several exciting new developments being reported over the last few years.

In this course, an overview of some of these recent developments will be presented. After an introduction to the general concepts of flow instability, presenting a range of examples, the major content of this course will be focussed on the broad class of flow instabilities where velocity "shear" and fluid inertia play key dynamical roles. Such flows, typically characterised by sufficiently "high" Reynolds number Ud/ν, where U and d are characteristic velocity and length scales of the flow, and ν is the kinematic viscosity of the fluid, are central to modelling flows in the environment and industry. They typically demonstrate the key role played by the redistribution of vorticity within the flow, and such vortical flow instabilities often trigger the complex, yet hugely important process of "transition to turbulence".

A hierarchy of mathematical approaches will be discussed to address a range of "stability" problems, from more traditional concepts of "linear" infinitesimal normal mode perturbation energy growth on laminar parallel shear flows to transient, inherently nonlinear perturbation growth of general measures of perturbation magnitude over finite time horizons where flow geometry and/or fluid properties play a dominant role. The course will also discuss in detail physical interpretations of the various flow instabilities considered, as well as the industrial and environmental application of the results of the presented mathematical analyses

Pre-requisites

Elementary concepts from undergraduate real analysis. Some knowledge of complex analysis would be advantageous (e.g. the level of IB Complex Analysis/Methods). No knowledge of functional analysis is assumed.

Contents

1 Linear stability analysis

1.1 Rayleigh–Taylor instability

1.2 Rayleigh–Bénard convection

1.3 Classical Kelvin–Helmholtz instability

1.4 Finite depth shear flow

1.5 Stratified flows

2 Absolute and convective instabilities

3 Transient growth

3.1 Motivation

3.2 A toy model

3.3 A general mathematical framework

3.4 Orr-Sommerfeld and Squire equations

4 A variational point of view