Part III — The Standard Model

Theorems with proof

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The Standard Model of particle physics is, by far, the most successful application of quantum field theory (QFT). At the time of writing, it accurately describes all experimental measurements involving strong, weak, and electromagnetic interactions. The course aims to demonstrate how this model, a QFT with gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ and fermion fields for the leptons and quarks, is realised in nature. It is intended to complement the more general Advanced QFT course.

We begin by defining the Standard Model in terms of its local (gauge) and global symmetries and its elementary particle content (spin-half leptons and quarks, and spin-one gauge bosons). The parity P, charge-conjugation C and time-reversal T transformation properties of the theory are investigated. These need not be symmetries manifest in nature; e.g. only left-handed particles feel the weak force and so it violates parity symmetry. We show how CP violation becomes possible when there are three generations of particles and describe its consequences.

Ideas of spontaneous symmetry breaking are applied to discuss the Higgs Mechanism and why the weakness of the weak force is due to the spontaneous breaking of the $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge symmetry. Recent measurements of what appear to be Higgs boson decays will be presented.

We show how to obtain cross sections and decay rates from the matrix element squared of a process. These can be computed for various scattering and decay processes in the electroweak sector using perturbation theory because the couplings are small. We touch upon the topic of neutrino masses and oscillations, an important window to physics beyond the Standard Model.

The strong interaction is described by quantum chromodynamics (QCD), the non-abelian gauge theory of the (unbroken) $\mathrm{SU}(3)$ gauge symmetry. At low energies quarks are confined and form bound states called hadrons. The coupling constant decreases as the energy scale increases, to the point where perturbation theory can be used. As an example we consider electron- positron annihilation to final state hadrons at high energies. Time permitting, we will discuss nonperturbative approaches to QCD. For example, the framework of effective field theories can be used to make progress in the limits of very small and very large quark masses.

Both very high-energy experiments and very precise experiments are currently striving to observe effects that cannot be described by the Standard Model alone. If time permits, we comment on how the Standard Model is treated as an effective field theory to accommodate (so far hypothetical) effects beyond the Standard Model.

Pre-requisites

It is necessary to have attended the Quantum Field Theory and the Symmetries, Fields and Particles courses, or to be familiar with the material covered in them. It would be advantageous to attend the Advanced QFT course during the same term as this course, or to study renormalisation and non-abelian gauge fixing.

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0 Introduction

1 Overview

2 Chiral and gauge symmetries

2.1 Chiral symmetry

Lemma. If ψ_L is left-handed and ϕ_R is right-handed, then

$$\bar{\psi}_L\phi_L = \bar{\psi}_R\phi_R = 0.$$

Proof. We only do the left-handed case.

$$\begin{aligned}\bar{\psi}_L\phi_L &= \psi_L^\dagger\gamma^0\phi_L\\ &= (P_L\psi_L)^\dagger\gamma^0(P_L\phi_L)\\ &= \psi_L^\dagger P_L\gamma^0 P_L\phi_L\\ &= \psi_L^\dagger P_L P_R\gamma^0\phi_L\\ &= 0,\end{aligned}$$

using the fact that $\{\gamma^5, \gamma^0\} = 0$. □

Proposition. If ψ is a massless Dirac spinor, then so are ψ_L and ψ_R.

Proposition. If we have a massless spinor u, then

$$hu(p) = \frac{\gamma^5}{2}u(p).$$

Proof. Note that if we have a massless particle, then we have

$$\not{p}u = 0,$$

since quantumly, p is just given by differentiation. We write this out explicitly to see

$$\gamma^\mu p_\mu u^s = (\gamma^0 p^0 - \boldsymbol{\gamma}\cdot\mathbf{p})u = 0.$$

Multiplying it by $\gamma^5\gamma^0/p^0$ gives

$$\gamma^5 u(p) = \gamma^5\gamma^0\gamma^i\frac{p^i}{p^0}u(p).$$

Again since the particle is massless, we know

$$(p^0)^2 - \mathbf{p}\cdot\mathbf{p} = 0.$$

So $\hat{\mathbf{p}} = \mathbf{p}/p^0$. Also, by direct computation, we find that

$$\gamma^5\gamma^0\gamma^i = 2S^i.$$

So it follows that

$$\gamma^5 u(p) = 2hu(p).$$ □

2.2 Gauge symmetry

3 Discrete symmetries

3.1 Symmetry operators

Theorem (Wigner's theorem)**.** Let $\mathcal{H}$ be a Hilbert space, and $f : \mathcal{H} \to \mathcal{H}$ be a bijection such that

- If $\Phi, \Psi \in \mathcal{H}$ differ by a phase, then $f\Phi$ and $f\Psi$ differ by a phase.
- For any $\Phi, \Psi \in \mathcal{H}$, we have

$$|\langle f\Phi, f\Psi\rangle| = |\langle \Phi, \Psi\rangle|.$$

Then there exists a map $W : \mathcal{H} \to \mathcal{H}$ that is either linear and unitary, or anti-linear and anti-unitary, such that for all $\Phi \in \mathcal{H}$, we have that $W\Phi$ and $f\Phi$ differ by a phase.

3.2 Parity

3.3 Charge conjugation

Proposition.

$$v^s(p) = C\bar{u}^{sT}, \quad u^s(p) = C\bar{v}^{sT}(p).$$

Proposition.

$$\begin{aligned}\psi^c(x) &\equiv \hat{C}\psi(x)\hat{C}^{-1} = \eta_c C\bar{\psi}^T(x)\\ \bar{\psi}^c(x) &\equiv \hat{C}\bar{\psi}(x)\hat{C}^{-1} = \eta_c^*\psi^T(x)C = -\eta_c^*\psi^T(x)C^{-1}.\end{aligned}$$

Proposition.

$$\begin{gathered}(C\gamma^\mu)^T = C\gamma^\mu\\ C = -C^T = -C^\dagger = -C^{-1}\\ (\gamma^\mu)^T = -C\gamma^\mu C^{-1}, \quad (\gamma^5)^T = +C\gamma^5 C^{-1}.\end{gathered}$$

3.4 Time reversal

3.5 S-matrix

3.6 CPT theorem

Theorem (CPT theorem)**.** Any Lorentz invariant Lagrangian with a Hermitian Hamiltonian should be invariant under the product of P, C and T.

3.7 Baryogenesis

4 Spontaneous symmetry breaking

4.1 Discrete symmetry

4.2 Continuous symmetry

4.3 General case

4.4 Goldstone's theorem

4.5 The Higgs mechanism

4.6 Non-abelian gauge theories

5 Electroweak theory

5.1 Electroweak gauge theory

5.2 Coupling to leptons

5.3 Quarks

5.4 Neutrino oscillation and mass

5.5 Summary of electroweak theory

6 Weak decays

6.1 Effective Lagrangians

6.2 Decay rates and cross sections

Proposition. Up to tree order, and a phase, we have

$$M_{fi} = \langle f| \mathcal{L}(0) |i\rangle ,$$

where $\mathcal{L}$ is the Lagrangian. We usually omit the (0).

Proof sketch. Up to tree order, we have

$$\langle f| S - 1 |i\rangle = i \int \mathrm{d}^4x \ \langle f| \mathcal{L}(x) |i\rangle$$

We write $\mathcal{L}$ in momentum space. Then the only x-dependence in the $\langle f| \mathcal{L}(x) |i\rangle$ factor is a factor of $e^{i(p_f - p_i)\cdot x}$. Integrating over x introduces a factor of $(2\pi)^4\delta^{(4)}(p_f - p_i)$. Thus, we must have had, up to tree order,

$$\langle f| \mathcal{L}(x) |i\rangle = M_{fi}e^{i(p_f - p_i)\cdot x}.$$

So evaluating this at $x = 0$ gives the desired result. □

6.3 Muon decay

6.4 Pion decay

6.5 K^0-$\bar{K}^0$ mixing

7 Quantum chromodynamics (QCD)

7.1 QCD Lagrangian

7.2 Renormalization

7.3 $e^+e^- \to$ hadrons

7.4 Deep inelastic scattering