Part III — The Standard Model
Definitions

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These notes are not endorsed by the lecturers, and I have modified them (often significantly) after lectures. They are nowhere near accurate representations of what was actually lectured, and in particular, all errors are almost surely mine.

The Standard Model of particle physics is, by far, the most successful application of quantum field theory (QFT). At the time of writing, it accurately describes all experimental measurements involving strong, weak, and electromagnetic interactions. The course aims to demonstrate how this model, a QFT with gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ and fermion fields for the leptons and quarks, is realised in nature. It is intended to complement the more general Advanced QFT course.

We begin by defining the Standard Model in terms of its local (gauge) and global symmetries and its elementary particle content (spin-half leptons and quarks, and spin-one gauge bosons). The parity P, charge-conjugation C and time-reversal T transformation properties of the theory are investigated. These need not be symmetries manifest in nature; e.g. only left-handed particles feel the weak force and so it violates parity symmetry. We show how CP violation becomes possible when there are three generations of particles and describe its consequences.

Ideas of spontaneous symmetry breaking are applied to discuss the Higgs Mechanism and why the weakness of the weak force is due to the spontaneous breaking of the $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge symmetry. Recent measurements of what appear to be Higgs boson decays will be presented.

We show how to obtain cross sections and decay rates from the matrix element squared of a process. These can be computed for various scattering and decay processes in the electroweak sector using perturbation theory because the couplings are small. We touch upon the topic of neutrino masses and oscillations, an important window to physics beyond the Standard Model.

The strong interaction is described by quantum chromodynamics (QCD), the non-abelian gauge theory of the (unbroken) $\mathrm{SU}(3)$ gauge symmetry. At low energies quarks are confined and form bound states called hadrons. The coupling constant decreases as the energy scale increases, to the point where perturbation theory can be used. As an example we consider electron- positron annihilation to final state hadrons at high energies. Time permitting, we will discuss nonperturbative approaches to QCD. For example, the framework of effective field theories can be used to make progress in the limits of very small and very large quark masses.

Both very high-energy experiments and very precise experiments are currently striving to observe effects that cannot be described by the Standard Model alone. If time permits, we comment on how the Standard Model is treated as an effective field theory to accommodate (so far hypothetical) effects beyond the Standard Model.

Pre-requisites

It is necessary to have attended the Quantum Field Theory and the Symmetries, Fields and Particles courses, or to be familiar with the material covered in them. It would be advantageous to attend the Advanced QFT course during the same term as this course, or to study renormalisation and non-abelian gauge fixing.

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1 Overview

2 Chiral and gauge symmetries

2.1 Chiral symmetry

Definition (Chirality)**.** A Dirac fermion ψ is *right-handed* if $\gamma^5\psi = \psi$, and *left-handed* if $\gamma^5\psi = -\psi$.

A left- or right-handed fermion is said to have *definite chirality*.

Notation.

$$\psi_L = P_L\psi, \quad \psi_R = P_R\psi.$$

Definition (Helicity)**.** We define the *helicity* to be the projection of the angular momentum onto the direction of the linear momentum:

$$h = \mathbf{J} \cdot \hat{\mathbf{p}} = \mathbf{S} \cdot \hat{\mathbf{p}},$$

where

$$\mathbf{J} = -i\mathbf{r} \times \nabla + \mathbf{S}$$

is the total angular momentum, and $\mathbf{S}$ is the spin operator given by

$$S_i = \frac{i}{4}\varepsilon_{ijk}\gamma^j\gamma^k = \frac{1}{2}\begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}.$$

2.2 Gauge symmetry

3 Discrete symmetries

Definition (Parity transform)**.** The *parity transform* is given by

$$\Lambda^\mu{}_\nu = \mathbb{P}^\mu{}_\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Definition (Time reversal transform)**.** The *time reversal transform* is given by

$$\mathbb{T}^\mu{}_\nu = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

3.1 Symmetry operators

Definition (Linear and anti-linear map)**.** Let $\mathcal{H}$ be a Hilbert space. A function $f : \mathcal{H} \to \mathcal{H}$ is *linear* if

$$f(\alpha\Phi + \beta\Psi) = \alpha f(\Phi) + \beta f(\Psi)$$

for all $\alpha, \beta \in \mathbb{C}$ and $\Phi, \Psi \in \mathcal{H}$. A map is *anti-linear* if

$$f(\alpha\Phi + \beta\Psi) = \alpha^* f(\Phi) + \beta^* f(\Psi).$$

Definition (Unitary and anti-unitary map)**.** Let $\mathcal{H}$ be a Hilbert space, and $f : \mathcal{H} \to \mathcal{H}$ a linear map. Then f is *unitary* if

$$\langle f\Phi, f\Psi\rangle = \langle \Phi, \Psi\rangle$$

for all $\Phi, \Psi \in \mathcal{H}$.

If $f : \mathcal{H} \to \mathcal{H}$ is anti-linear, then it is *anti-unitary* if

$$\langle f\Phi, f\Psi\rangle = \langle \Phi, \Psi\rangle^*.$$

3.2 Parity

Definition (Intrinsic parity)**.** The *intrinsic parity* of a field ϕ is the number $\eta_P \in \mathbb{C}$ such that

$$\hat{P}\phi(x)\hat{P}^{-1} = \eta_P \phi(x_P).$$

Definition (Scalar and pseudoscalar fields)**.** A real scalar field is called a *scalar field* (confusingly) if the intrinsic parity is -1. Otherwise, it is called a *pseudoscalar field.*

Definition (Vector and axial vector fields)**.** *Vector fields* are vector fields with $\eta_P = -1$. Otherwise, they are *axial vector fields.*

3.3 Charge conjugation

3.4 Time reversal

3.5 S-matrix

3.6 CPT theorem

3.7 Baryogenesis

4 Spontaneous symmetry breaking

4.1 Discrete symmetry

4.2 Continuous symmetry

4.3 General case

4.4 Goldstone's theorem

4.5 The Higgs mechanism

4.6 Non-abelian gauge theories

5 Electroweak theory

5.1 Electroweak gauge theory

Definition (Higgs field)**.** The *Higgs field* ϕ is a complex scalar field with two components, $\phi(x) \in \mathbb{C}^2$. The SU(2) action is given by the fundamental representation on $\mathbb{C}^2$, and the hypercharge is $Y = \frac{1}{2}$.

Explicitly, an (infinitesimal) gauge transformation can be represented by elements $\alpha^a(x), \beta(x) \in \mathbb{R}$, corresponding to the elements $\alpha^a(x)\tau^a \in \mathfrak{su}(2)$ and $\beta(x) \in \mathfrak{u}(1) \cong \mathbb{R}$. Then the Higgs field transform as

$$\phi(x) \mapsto e^{i\alpha^a(x)\tau^a} e^{i\frac{1}{2}\beta(x)}\phi(x),$$

where the $\frac{1}{2}$ factor of $\beta(x)$ comes from the hypercharge being $\frac{1}{2}$.

5.2 Coupling to leptons

5.3 Quarks

5.4 Neutrino oscillation and mass

5.5 Summary of electroweak theory

6 Weak decays

6.1 Effective Lagrangians

6.2 Decay rates and cross sections

Definition (Decay rate)**.** Let X be a particle. The decay rate Γ_X is rate of decay of X in its rest frame. In other words, if we have a sample of X, then this is the number of decays of X per unit time divided by the number of X present.

The *lifetime* of X is

$$\tau \equiv \frac{1}{\Gamma_X}.$$

We can write

$$\Gamma_X = \sum_{f_i} \Gamma_{X \to f_i},$$

where $\Gamma_{X \to f_i}$ is the partial decay rate to the final state f_i.

Definition (Invariant amplitude)**.** We define the *invariant amplitude* M by

$$\langle f | S - 1 | i \rangle = (2\pi)^4 \delta^{(4)}(p_f - p_i) i M_{fi}.$$

Definition (Cross section)**.** The *cross section* is defined by

$$n = F\sigma.$$

6.3 Muon decay

6.4 Pion decay

6.5 K^0-$\bar{K}^0$ mixing

7 Quantum chromodynamics (QCD)

7.1 QCD Lagrangian

7.2 Renormalization

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