Part III — Classical and Quantum Solitons
Definitions

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Solitons are solutions of classical field equations with particle-like properties. They are localised in space, have finite energy and are stable against decay into radiation. The stability usually has a topological explanation. After quantisation, they give rise to new particle states in the underlying quantum field theory that are not seen in perturbation theory. We will focus mainly on kink solitons in one space dimension, on gauge theory vortices in two dimensions, and on Skyrmions in three dimensions.

Pre-requisites

This course assumes you have taken Quantum Field Theory and Symmetries, Fields and Particles. The small amount of topology that is needed will be developed during the course.

Reference

N. Manton and P. Sutcliffe, *Topological Solitons*, CUP, 2004

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1 ϕ^4 kinks

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Definition (Ginzburg–Landau vortex)**.** A global *Ginzburg–Landau vortex* of charge $N > 0$ is a (smooth) solution of the ungauged Ginzburg–Landau equation of the form

$$\Phi = f_N(r)e^{iN\theta}$$

in polar coordinates (r, θ). Moreover, we require that $f_N(r) \to 1$ as $r \to \infty$.

2.3 Abelian Higgs/Gauged Ginzburg–Landau vortices

Definition (Magnetic field/curvature)**.** The *magnetic field*, or *curvature* is given by

$$B = \partial_1 A_2 - \partial_2 A_1.$$

We can alternatively think of it as the 2-form

$$F = \mathrm{d}A = B\ \mathrm{d}x^1 \wedge \mathrm{d}x^2.$$

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