

# Part III — Classical and Quantum Solitons

## Definitions

Based on lectures by N. S. Manton and D. Stuart

Notes taken by Dexter Chua

Easter 2017

Solitons are solutions of classical field equations with particle-like properties. They are localised in space, have finite energy and are stable against decay into radiation. The stability usually has a topological explanation. After quantisation, they give rise to new particle states in the underlying quantum field theory that are not seen in perturbation theory. We will focus mainly on kink solitons in one space dimension, on gauge theory vortices in two dimensions, and on Skyrmions in three dimensions.

### **Pre-requisites**

This course assumes you have taken Quantum Field Theory and Symmetries, Fields and Particles. The small amount of topology that is needed will be developed during the course.

### **Reference**

N. Manton and P. Sutcliffe, *Topological Solitons*, CUP, 2004

## Contents

<b>0</b>	<b>Introduction</b>	<b>3</b>
<b>1</b>	<b><math>\phi^4</math> kinks</b>	<b>4</b>
1.1	Kink solutions . . . . .	4
1.2	Dynamic kink . . . . .	4
1.3	Soliton interactions . . . . .	4
1.4	Quantization of kink motion . . . . .	4
1.5	Sine-Gordon kinks . . . . .	4
<b>2</b>	<b>Vortices</b>	<b>5</b>
2.1	Topological background . . . . .	5
2.2	Global $U(1)$ Ginzburg–Landau vortices . . . . .	5
2.3	Abelian Higgs/Gauged Ginzburg–Landau vortices . . . . .	5
2.4	Bogomolny/self-dual vortices and Taubes’ theorem . . . . .	5
2.5	Physics of vortices . . . . .	5
2.6	Jackiw–Pi vortices . . . . .	5
<b>3</b>	<b>Skyrmions</b>	<b>6</b>
3.1	Skyrme field and its topology . . . . .	6
3.2	Skyrmion solutions . . . . .	6
3.3	Other Skyrmion structures . . . . .	6
3.4	Asymptotic field and forces for $B = 1$ hedgehogs . . . . .	6
3.5	Fermionic quantization of the $B = 1$ hedgehog . . . . .	6
3.6	Rigid body quantization . . . . .	6

## 0 Introduction

## 1 $\phi^4$ kinks

### 1.1 Kink solutions

### 1.2 Dynamic kink

### 1.3 Soliton interactions

### 1.4 Quantization of kink motion

### 1.5 Sine-Gordon kinks

## 2 Vortices

### 2.1 Topological background

### 2.2 Global $U(1)$ Ginzburg–Landau vortices

**Definition** (Ginzburg–Landau vortex). A global *Ginzburg–Landau vortex* of charge  $N > 0$  is a (smooth) solution of the ungauged Ginzburg–Landau equation of the form

$$\Phi = f_N(r)e^{iN\theta}$$

in polar coordinates  $(r, \theta)$ . Moreover, we require that  $f_N(r) \rightarrow 1$  as  $r \rightarrow \infty$ .

### 2.3 Abelian Higgs/Gauged Ginzburg–Landau vortices

**Definition** (Magnetic field/curvature). The *magnetic field*, or *curvature* is given by

$$B = \partial_1 A_2 - \partial_2 A_1.$$

We can alternatively think of it as the 2-form

$$F = dA = B dx^1 \wedge dx^2.$$

### 2.4 Bogomolny/self-dual vortices and Taubes' theorem

### 2.5 Physics of vortices

### 2.6 Jackiw–Pi vortices

### **3 Skyrmions**

**3.1 Skyrme field and its topology**

**3.2 Skymion solutions**

**3.3 Other Skymion structures**

**3.4 Asymptotic field and forces for  $B = 1$  hedgehogs**

**3.5 Fermionic quantization of the  $B = 1$  hedgehog**

**3.6 Rigid body quantization**